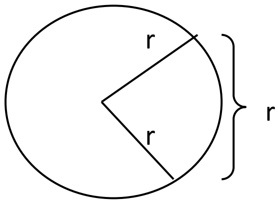
1.0 CONCEPTS & FORMULAS

1.1 INTRODUCTION

Radian

The angle subtended at centre of a circle by an arc of length equal to the radius of the circle is 1 radian



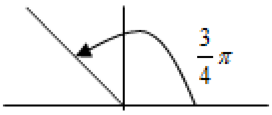
$1^o = \dfrac{\pi}{180^o}$ radian

1 radian = $\dfrac{180^o}{\pi}$

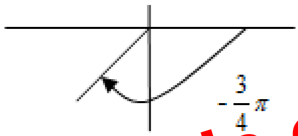
$1^o = 60'$

Positive & Negative Angles

A positive angle is measured in an anticlockwise direction

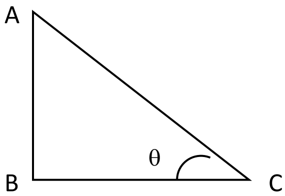


A negative angle is measured in a clockwise direction



1.2 TRIGONOMETRIC FUNCTIONS CHARACTERISTICS

Six Trigonometric Functions



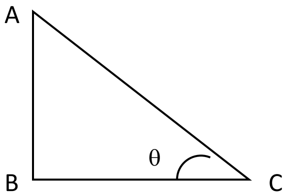
$\sin\theta = \dfrac{AB}{AC}$; $\cos\theta = \dfrac{BC}{AC}$; $\tan\theta = \dfrac{AB}{BC}$

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{AC}{AB}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{AC}{BC}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{BC}{AB}$

Important Trigonometric Ratios



θ	Sine	Cosine	Tangent
0	0	1	0
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	1	0	∞

Sign of Trigonometric Functions

Sine positive *Cos negative* *Tan negative*	Quadrant 2	Quadrant 1	*Sine positive* *Cos positive* *Tan positive*
Sine negative *Cos negative* *Tan positive*	Quadrant 3	Quadrant 4	*Sine negative* *Cos positive* *Tan negative*

Trigonometry Identities & Formulas

Basic Identities :

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta = 1 + \tan^2\theta$$

$$\mathrm{cosec}^2\theta = 1 + \cot^2\theta$$

Compound-Angle Formulas :

sin(A + B) = sin A cos B + cos A sin B
sin(A – B) = sin A cos B - cos A sin B
cos(A + B) = cos A cos B – sin A sin B
cos (A - B) = cos A cos B + sin A sin B

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Double-Angle Formulas :

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 1 - 2\sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Factor Formulas (Sum to Product) :

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos D - \cos C = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

Factor Formulas (Product to Sum) :

2 sin A cos B = sin (A + B) + sin (A – B)
2 cos A sin B = sin (A + B) - sin (A – B)
2 cos A cos B = cos (A + B) + cos (A – B)
2 sin A sin B = cos (A – B) – cos (A + B)

11.	Solve each of the following equations giving all solutions in the given interval

Suggested Solution

a) $25\cos(\theta - 73.74) = 15$, for $0^0 \le \theta \le 360^0$.

Suggested Solution

$$25\cos(\theta - 73.74) = 15$$
$$\cos(\theta - 73.74) = 0.6$$
$$\theta - 73.74^0 = 53.13^0, 306.87^0$$
$$\theta = 53.13^0 + 73.74^0,$$
$$(306.87^0 + 73.74^0 - 360^0)$$
$$\theta = \underline{\underline{20.6^0, 126.9^0}}\ (Answer)$$

b) $17\sin(\theta - 61.93) = 14$, for $0^0 \le \theta \le 360^0$.

Suggested Solution

$$17\sin(\theta - 61.93) = 14$$
$$\sin(\theta - 61.93) = \frac{14}{17}$$
$$\theta - 61.93 = 55.44,)180 - 55.44)$$
$$\theta = 55.44 + 61.93, ,124.56 + 61.93$$
$$\theta = \underline{\underline{117.4^0, 186.5^0}}\ \ (Answer)$$

c) $13\sin(x + 67.38) = 11$, for $0^0 < \theta < 180^0$

Suggested Solution

$$13\sin(x + 67.38) = 11$$
$$\sin(x + 67.38) = \frac{11}{13}$$
$$x + 67.38 = 57.8, 122.2$$
$$x = -9.58, 54.82$$
$$2\theta = 54.82, (360 - 9.58)$$
$$2\theta = 54.82, 350.42$$
$$\theta = \underline{\underline{27.4^0, 175.2^0}}\ \ (Answer)$$

Note

Since $0^0 < \theta < 180^0$ then $0^0 < 2\theta < 360^0$

d) $\sqrt{26}\cos(\theta + 11.31) = 4$, for $0^0 \le \theta \le 360^0$

Suggested Solution

$$\sqrt{26}\cos(\theta + 11.31) = 4$$
$$\cos(\theta + 11.31) = \frac{4}{\sqrt{26}}$$
$$\theta + 11.31 = 38.33,\ 321.67$$
$$\theta = \underline{\underline{27.0^0, 310.4^0}}\ \ (Answer)$$

12.	Given that $\tan\beta = 2$. Find without the use of a calculator the exact value of $\tan\alpha$, given that a) $\tan(\alpha + \beta) = 4$ b) $\sin(\alpha + \beta) = 3\cos(\alpha - \beta)$

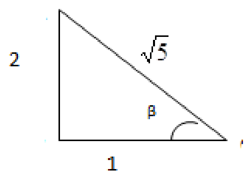
Suggested Solution

a)

$$\tan(\alpha + \beta) = 4$$
$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 4$$
$$\tan\alpha + 2 = 4(1 - 2\tan\alpha)$$
$$\tan\alpha + 2 = 4 - 8\tan\alpha$$
$$9\tan\alpha = 2$$
$$\tan\alpha = \underline{\underline{\frac{2}{9}}}\ (Answer)$$

b)

$$\sin(\alpha + \beta) = 3\cos(\alpha - \beta)$$
$$\sin\alpha\cos\beta + \cos\alpha\sin\beta = 3(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$
$$\frac{1}{\sqrt{5}}\sin\alpha + \frac{2}{\sqrt{5}}\cos\alpha = 3\left(\frac{1}{\sqrt{5}}\cos\alpha + \frac{2}{\sqrt{5}}\sin\alpha\right)$$
$$\frac{\sin\alpha + 2\cos\alpha}{\sqrt{5}} = \frac{3\cos\alpha + 6\sin\alpha}{\sqrt{5}}$$
$$-5\sin\alpha = \cos\alpha$$
$$\tan\alpha = \underline{\underline{-\frac{1}{5}}}\ (Answer)$$



$$\tan\beta = 2$$
$$\sin\beta = \frac{2}{\sqrt{5}}$$
$$\cos\beta = \frac{1}{\sqrt{5}}$$

16	Prove the identity $\cos 3x \equiv 4\cos^3 x - 3\cos x$

Suggested Solution

$$= \cos 3x$$
$$= \cos(2x + x)$$
$$= \cos 2x \cos - \sin 2x \sin x \quad \text{(Note)}$$
$$= (2\cos^2 x - 1)\cos x - (2\sin x \cos x)(\sin x)$$
$$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x$$
$$= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x)$$
$$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$$
$$= 4\cos^3 x - 3\cos x \; (Shown)$$

Note: *Use Compound-Angle Formula*
cos(A + B) = cos A cos B – sin A sin B

17.	Prove the identity $\sin 3x = 3\sin x - 4\sin^3 x$

Suggested Solution

$$= \sin 3x$$
$$= \sin(2x + x)$$
$$= \sin 2x \cos x + \cos 2x \sin x \quad \text{(Note)}$$
$$= 2\sin x \cos x \cos x + (1 - 2\sin^2 x)\sin x$$
$$= 2\sin x \cos^2 x + \sin x - 2\sin^3 x$$
$$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$$
$$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$$
$$= 3\sin x - 4\sin^3 x \; (Shown)$$

Note: *Use Compound-Angle Formula*
sin(A + B) = sin A cos B + cos A sin B

18.	Prove the identity $\cos 4x + 4\cos 2x \equiv 8\cos^4 x - 3$

Suggested Solution

$$= \cos 4x + 4\cos 2x$$
$$= \cos(2x + 2x) + 4(2\cos^2 x - 1)$$
$$= \cos 2x \cos 2x - \sin 2x \sin 2x + 8\cos^2 x - 4$$
$$= (2\cos^2 x - 1)(2\cos^2 x - 1) - (2\sin x \cos x . 2\sin x \cos x) + 8\cos^2 x - 4$$
$$= 4\cos^4 x - 4\cos^2 x + 1 - 4\sin^2 x \cos^2 x + 8\cos^2 x - 4$$
$$= 4\cos^4 x - 4\cos^2 x + 1 - 4(1 - \cos^2 x)(\cos^2 x) + 8\cos^2 x - 4$$
$$= 4\cos^4 x - 4\cos^2 x + 1 - 4\cos^2 x + 4\cos^4 x + 8\cos^2 x - 4$$
$$= 8\cos^4 x - 3 \; (Shown)$$

19.	Prove the identity $\sin^2 x \cos^2 x \equiv \frac{1}{8}(1 - \cos 4x)$

Suggested Solution

$$= \sin^2 x \cos^2 x$$
$$= \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{\cos 2x + 1}{2}\right) \quad (Note)$$
$$= \frac{\cos 2x + 1 - \cos^2 2x - \cos 2x}{4}$$
$$= \frac{1 - \cos^2 2x}{4} \quad \Rightarrow \text{Let } 2x = A$$
$$= \frac{1 - \cos^2 A}{4} = \frac{1 - \left(\frac{\cos 2A + 1}{2}\right)}{4} = \frac{2 - \cos 2A - 1}{8} = \frac{1 - \cos 2A}{8} = \frac{1 - \cos 4x}{8}$$
$$= \frac{1}{8}(1 - \cos 4x) \; (Shown)$$

Note

$\cos 2x = 2\cos^2 - 1$ $2\cos^2 x = \cos 2x + 1$ $\cos^2 x = \frac{\cos 2x + 1}{2}$
$\cos 2x = 1 - 2\sin^2 x$ $2\sin^2 x = 1 - \cos 2x$ $\sin^2 x = \frac{1 - \cos 2x}{2}$

24. Express $8\sin\theta - 15\cos\theta$ in the form $R\sin(\theta-\alpha)$, where $R>0$ and $0^0 < \alpha < 90^0$, giving the exact value of R and the value of α. correct to 2 decimal places

Suggested Solution

$$\text{Let } 8\sin\theta + 15\cos\theta = R\sin(\theta-\alpha)$$

$$8\sin\theta + 15\cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$$

Equate the coefficients of $\cos\theta$ and $\sin\theta$

$$\Rightarrow R\cos\alpha = 8 \text{........................(1)}$$

$$\Rightarrow -R\sin\alpha = 15 \text{........................(2)}$$

$$\Rightarrow (1)^2 + (2)^2 :$$

$$\Rightarrow \{R^2\cos^2\alpha = 64\} + \{R^2\sin^2\alpha = 225\}$$

$$\Rightarrow R^2 = 64 + 225 = 289 \Rightarrow \underline{\underline{R = 17}}\ (Answer)$$

$$\Rightarrow \text{Replace } R \text{ in } (1):$$

$$\Rightarrow \cos\alpha = \frac{8}{17}$$

$$\Rightarrow \alpha = \cos^{-1}\frac{8}{17} = \underline{\underline{61.93^0}}\ (Answer)$$

25. Express $5\sin x + 12\cos x$ in the form $R\sin(x+\alpha)$, where $R>0$ and $0^0 < \alpha < 90^0$, giving the value of α correct to 2 decimal places.

Suggested Solution

$$\text{Let } 5\sin x + 12\cos x = R\sin(x+\alpha)$$

$$5\sin x + 12\cos x = R\sin x\cos\alpha + R\cos x\sin\alpha$$

Equate the coefficients of cosx and sinx

$$\Rightarrow R\cos\alpha = 5 \text{........................(1)}$$

$$\Rightarrow R\sin\alpha = 12 \text{........................(2)}$$

$$\Rightarrow (1)^2 + (2)^2 :$$

$$\Rightarrow \{R^2\cos^2\alpha = 25\} + \{R^2\sin^2\alpha = 144\}$$

$$\Rightarrow R^2 = 25 + 144 = 169 \Rightarrow \underline{\underline{R = 13}}\ (Answer)$$

$$\Rightarrow \text{Replace } R \text{ in } (1):$$

$$\Rightarrow \cos\alpha = \frac{5}{13}$$

$$\Rightarrow \alpha = \cos^{-1}\frac{5}{13} = \underline{\underline{67.38^0}}\ (Answer)$$

26. Express $5\cos\theta - \sin\theta$ in the form $R\cos(\theta+\alpha)$, where $R>0$ and $0^0 < \alpha < 90^0$, giving the exact value of R and the value of α correct to 2 decimal places.

Suggested Solution

$$\text{Let } 5\cos\theta - \sin\theta = R\cos(\theta+\alpha)$$

$$5\cos\theta - \sin\theta = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$$

Equate the coefficients of $\cos\theta$ and $\sin\theta$

$$\Rightarrow R\cos\alpha = 5 \text{........................(1)}$$

$$\Rightarrow R\sin\alpha = 1 \text{........................(2)}$$

$$\Rightarrow (1)^2 + (2)^2 :$$

$$\Rightarrow \{R^2\cos^2\alpha = 25\} + \{R^2\sin^2\alpha = 1\}$$

$$\Rightarrow R^2 = 25 + 1 = 26 \Rightarrow \underline{\underline{R = \sqrt{26}}}\ (Answer)$$

$$\Rightarrow \text{Replace } R \text{ in } (1):$$

$$\Rightarrow \cos\alpha = \frac{5}{\sqrt{26}}$$

$$\Rightarrow \alpha = \cos^{-1}\frac{5}{\sqrt{26}} = \underline{\underline{11.31^0}}\ (Answer)$$

27. Express $9\sin\theta - 12\cos\theta$ in the form $R\sin(\theta-\alpha)$, where $R>0$ and $0^0 < \alpha < 90^0$, giving the exact value of R and the value of α correct to 2 decimal places..

Suggested Solution

$$\text{Let } 9\sin\theta - 12\cos\theta = R\sin(\theta-\alpha)$$

$$9\sin\theta - 12\cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$$

Equate the coefficients of $\cos\theta$ and $\sin\theta$

$$\Rightarrow R\cos\alpha = 9 \text{........................(1)}$$

$$\Rightarrow R\sin\alpha = 12 \text{........................(2)}$$

$$\Rightarrow (1)^2 + (2)^2 :$$

$$\Rightarrow \{R^2\cos^2\alpha = 81\} + \{R^2\sin^2\alpha = 144\}$$

$$\Rightarrow R^2 = 81 + 144 = 225 \Rightarrow \underline{\underline{R = 15}}\ (Answer)$$

$$\Rightarrow \text{Replace } R \text{ in } (1):$$

$$\Rightarrow \cos\alpha = \frac{9}{15}$$

$$\Rightarrow \alpha = \cos^{-1}\frac{9}{15} = \underline{\underline{53.13^0}}\ (Answer)$$

28. Prove the identity

$$\sec^2 x + \sec x \tan x \equiv \frac{1}{1-\sin x}$$

Suggested Solution

$$\Rightarrow \sec^2 x + \sec x \tan x$$

$$\Rightarrow \frac{1}{\cos^2 x} + \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$$

$$\Rightarrow \frac{1+\sin x}{\cos^2 x}$$

$$\Rightarrow \frac{1+\text{sinx}}{1-\sin^2 x}$$

$$\Rightarrow \frac{1+\sin x}{(1+\sin x)(1-\sin x)}$$

$$\Rightarrow \frac{1}{1-\sin x} \quad (\textit{Shown})$$

29. Prove the identity

$$\cot x - \cot 2x \equiv \operatorname{cosec} 2x$$

Suggested Solution

$$= \cot x - \cot 2x$$

$$= \frac{1}{\tan x} - \frac{1}{\tan 2x}$$

$$= \frac{1}{\tan x} - \frac{1-\tan^2 x}{2\tan x}$$

$$= \frac{2\tan x - \tan x(1-\tan^2 x)}{2\tan^2 x}$$

$$= \frac{2\tan x - \tan x + \tan^3 x}{2\tan^2 x}$$

$$= \frac{\tan x + \tan^3 x}{2\tan^2 x}$$

$$= \frac{1+\tan^2 x}{2\tan x} \; (\textit{Shown})$$

30. Show that $\tan^2 x + \cos^2 x \equiv \sec^2 x + \frac{1}{2}\cos 2x - \frac{1}{2}$

Suggested Solution

$$= \tan^2 x + \cos^2 x$$

$$= \sec^2 x - 1 + \cos^2 x$$

$$= \sec^2 x - 1 + \frac{\cos 2x + 1}{2}$$

$$= \sec^2 x - 1 + \frac{1}{2}\cos 2x + \frac{1}{2}$$

$$= \sec^2 x + \frac{1}{2}\cos 2x - \frac{1}{2} \; (\textit{Shown})$$

31. Show that, $\frac{1-4\sin^4 x}{1-4\cos^4 x}$ is negative for all values of x.

Suggested Solution

$$\Rightarrow \frac{1-4\sin^4 x}{1-4\cos^4 x}$$

$$\Rightarrow \frac{(1+2\sin^2 x)(1-2\sin^2 x)}{(1+2\cos^2 x)(1-2\cos^2 x)}$$

$$\Rightarrow \frac{(1-\cos 2x)\cos 2x}{(\cos 2x+1)(-\cos 2x)} \qquad \Rightarrow \begin{matrix} 2\sin^2 x = \cos 2x - 1 \\ 2\cos^2 x = \cos 2x + 1 \end{matrix}$$

$$\Rightarrow -\frac{(1-\cos 2x)}{1+\cos 2x}$$

$$\Rightarrow -\frac{(1-(2\cos^2 x - 1))}{1+(2\cos^2 x - 1)}$$

$$\Rightarrow -\frac{2-2\cos^2 x}{2\cos^2 x}$$

$$\Rightarrow -\frac{2(1-\cos^2 x)}{2\cos^2 x}$$

$$\Rightarrow -\frac{\sin^2 x}{\cos^2 x}$$

$$\Rightarrow -\tan^2 x$$

Since $\tan^2 x > 0$ for all x,

$$\therefore \frac{1-4\sin^4 x}{1-4\cos^4 x} < 0 \text{ for all } x \; (\textit{Shown})$$

35.	a)	Prove that $\sin^2 2\theta\left(\operatorname{cosec}^2\theta - \sec^2\theta\right) = 4\cos 2\theta$
	b)	i) Solve for $0^0 \le \theta \le 180^0$ the equation $\sin^2 2\theta\left(\operatorname{cosec}^2\theta - \sec^2\theta\right) = 3$
		ii) Find the exact value of $\operatorname{cosec}^2 15^0 - \sec^2 15^0$

Suggested Solution

a)

$$= \sin^2 2\theta\left(\cos ec^2\theta - \sec^2\theta\right)$$

$$= \sin 2\theta . \sin 2\theta\left(\frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta}\right)$$

$$= (2\sin\theta\cos\theta)(2\sin\theta\cos\theta)\left(\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta\cos^2\theta}\right)$$

$$= 4\sin^2\theta\cos^2\theta\left(\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta\cos^2\vartheta}\right)$$

$$= 4\left(\cos^2\theta - \sin^2\theta\right)$$

$$= 4\cos 2\theta \ (Shown)$$

b) i)

$$4\cos 2\theta = 3$$

$$\cos 2\theta = 0.75$$

$$2\theta = 41.4, 318.6$$

$$\theta = \underline{\underline{20.7^0,\ 159.3^0}} \ (Answer)$$

ii)

$$\sin^2 2\theta\left(\cos ec^2\theta - \sec^2\theta\right) = 4\cos 2\theta$$

$$\cos ec^2\theta - \sec^2\theta = \frac{4\cos 2\theta}{\sin^2 2\theta}$$

$$\cos ec^2 15^0 - \sec^2 15^0 = \frac{4\cos 30^0}{\sin^2 30^0}$$

$$= \frac{4\times\frac{\sqrt{3}}{2}}{\frac{1}{4}}$$

$$= \underline{\underline{8\sqrt{3}}} \ (Answer)$$

36.	i)	Show that the equation $\tan(60^0 + \theta) + \tan(60^0 - \theta) = k$ can be written in the form $\left(2\sqrt{3}\right)(1 + \tan^2\theta) = k(1 - 3\tan^2\theta)$
	ii)	Hence solve the equation $\tan(60^0 + \theta) + \tan(60^0 - \theta) = 3\sqrt{3}$ giving all solutions in the interval $0^0 \le \theta \le 180^0$

Suggested Solution

i)

$$\Rightarrow \tan(60^0 + \theta) + \tan(60^0 - \theta) = k$$

$$\Rightarrow \frac{\tan 60 + \tan\theta}{1 - \tan 60\tan\theta} + \frac{\tan 60 - \tan\theta}{1 + \tan 60\tan\theta} = k$$

$$\Rightarrow \frac{\sqrt{3} + \tan\theta}{1 - \sqrt{3}\tan\theta} + \frac{\sqrt{3} - \tan\theta}{1 + \sqrt{3}\tan\theta} = k$$

$$\Rightarrow \frac{(\sqrt{3} + \tan\theta)(1 + \sqrt{3}\tan\theta) + (\sqrt{3} - \tan\theta)(1 - \sqrt{3}\tan\theta)}{(1 - \sqrt{3}\tan\theta)(1 + \sqrt{3}\tan\theta)} = k$$

$$\Rightarrow \frac{2\sqrt{3} + 2\sqrt{3}\tan^2\theta}{1 - 3\tan^2\theta} = k$$

$$\Rightarrow \underline{\underline{2\sqrt{3}(1 + \tan^2\theta) = k(1 - 3\tan^2\theta)}} \ (Shown)$$

ii)

$$\tan(60^0 + \theta) + \tan(60^0 - \theta) = 3\sqrt{3}$$

From part(i), $k = 3\sqrt{3}$

$$\Rightarrow 2\sqrt{3}(1 + \tan^2\theta) = 3\sqrt{3}(1 - 3\tan^2\theta)$$

$$\Rightarrow 2(1 + \tan^2\theta) = 3(1 - 3\tan^2\theta)$$

$$\Rightarrow 2 + 2\tan^2\theta = 3 - 9\tan^2\theta$$

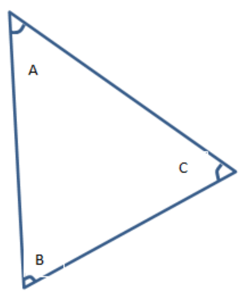
$$\Rightarrow 11\tan^2\theta = 1$$

$$\Rightarrow \tan^2\theta = \frac{1}{11}$$

$$\Rightarrow \tan\theta = \pm\sqrt{\frac{1}{11}}$$

$$\Rightarrow \theta = \underline{\underline{16.8^0, 163.2^0}} \ (Answer)$$

37.	Given the following triangle. Prove that $\tan A + \tan B + \tan C = \tan A\tan B\tan C$,



Suggested Solution

$$\Rightarrow A + B + C = 180$$

$$\Rightarrow A = 180 - (B + C)$$

$$\Rightarrow \tan A = \frac{\tan 180 - \tan(B + C)}{1 - \tan 180\tan(B + C)}$$

$$\Rightarrow \tan A = \frac{0 - \tan(B + C)}{1}$$

$$\Rightarrow \tan A = -\tan(B + C)$$

$$\Rightarrow \tan A = -\left[\frac{\tan B + \tan C}{1 - \tan B\tan C}\right]$$

$$\Rightarrow \tan A(1 - \tan B\tan C) = -\tan B - \tan C$$

$$\Rightarrow \tan A - \tan A\tan B\tan C = -\tan B - \tan C$$

$$\Rightarrow \tan A + \tan B + \tan C = \tan A\tan B\tan C \ (Shown)$$