Definition: A model M of basic modal logic is specified by three things:

1. A set W, whose elements are called worlds;
2. A relation R on W ($R \subseteq W \times W$), called the accessibility relation;
3. A function $L : W \rightarrow P(Atoms)$, called the labelling function.

We write R(x, y)to denote that (x, y) is in R.

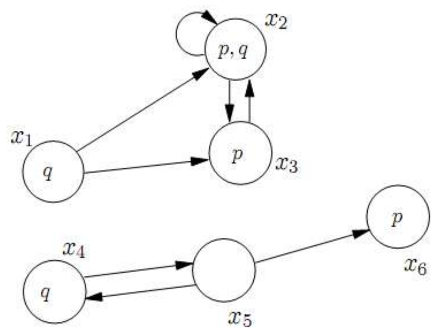
These models are often called Kripke models. Intuitively, $w \in W$ stands for a possible world and R(w,w') means that w' is a world accessible from world w. Suppose W equals $\{x_1,x_2,x_3,x_4,x_5,x_6\}$ and the relation R is given as follows:

$$R(x_1,x_2), R(x_1,x_3), R(x_2,x_2), R(x_2,x_3), R(x_3,x_2), R(x_4,x_5), R(x_5,x_4), R(x_5,x_6)$$

Suppose further that the labelling function behaves as follows:

x	x_1	x_2	x_3	x_4	x_5	x_6
$L(x)$	$\{q\}$	$\{p,q\}$	$\{p\}$	$\{q\}$	$\emptyset$	$\{p\}$

Then, the Kripke model is illustrated in Figure given below:



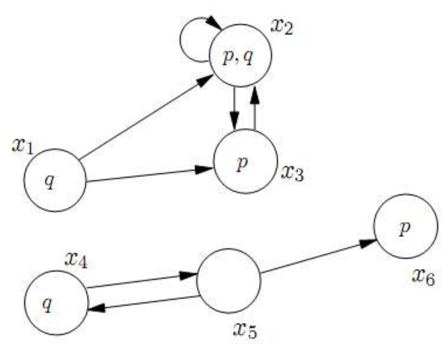
Definition: Let M=(W, R, L)bea model of basic modal logic. Suppose $x \in W$ and φ is a formula of modal logic. The φ is true in the world x when $x \Vdash \varphi$ (satisfaction relation).

$$
\begin{aligned}
x &\Vdash \top \\
x &\nVdash \bot \\
x &\Vdash p \quad \text{iff } p \in L(x) \\
x &\Vdash \neg\phi \quad \text{iff } x \nVdash \phi \\
x &\Vdash \phi \wedge \psi \quad \text{iff } x \Vdash \phi \text{ and } x \Vdash \psi \\
x &\Vdash \phi \vee \psi \quad \text{iff } x \Vdash \phi, \text{ or } x \Vdash \psi \\
x &\Vdash \phi \rightarrow \psi \quad \text{iff } x \Vdash \psi, \text{ whenever we have } x \Vdash \phi \\
x &\Vdash \phi \leftrightarrow \psi \quad \text{iff } (x \Vdash \phi \text{ iff } x \Vdash \psi) \\
x &\Vdash \Box\psi \quad \text{iff, for each } y \in W \text{ with } R(x,y), \text{ we have } y \Vdash \psi \\
x &\Vdash \Diamond\psi \quad \text{iff there is a } y \in W \text{ such that } R(x,y) \text{ and } y \Vdash \psi.
\end{aligned}
$$

For $\Box\varphi$ to be true at x, we require that φ be true in all the worlds accessible by R from x. For $\Diamond\varphi$, it is required that there is at least one accessible world in which φ is true. The meaning of $\varphi_1 \leftrightarrow \varphi_2$ coincides with that of $(\varphi_1 \rightarrow \varphi_2) \wedge (\varphi_2 \rightarrow \varphi_1)$.

Definition: A model M=(W, R, L) of basic modal logic is said to satisfy a formula if every state in the model satisfies it. Thus, we write $M \vDash \varphi$ iff, for each $x \in W$, $x \Vdash \varphi$.

Example: Consider the diagram given below:



Logic Engineering

The readings of ◊ corresponding to each reading of □ are given as follow:

$\Box\phi$	$\Diamond\phi$
It is necessarily true that ϕ	It is possibly true that ϕ
It will always be true that ϕ	Sometime in the future ϕ
It ought to be that ϕ	It is permitted to be that ϕ
Agent Q believes that ϕ	ϕ is consistent with Q's beliefs
Agent Q knows that ϕ	For all Q knows, ϕ
After any execution of program P, ϕ holds	After some execution of P, ϕ holds

Part of the job of logic engineering is to determine what formula schemes should be valid. The readings related to □ are given below:

- Necessity
- Always in future
- Ought
- Belief
- Knowledge
- Execution of programs

We canalso engineer logics at the level of Kripke models. For each of our six readings of □, there is a corresponding reading of the accessibility relation R which will then suggest that R enjoys certain properties such as reflexivity or transitivity. Let us start with necessity. The clauses

$$
\begin{array}{ll}
x \Vdash \Box\psi & \text{iff for each } y \in W \text{ with } R(x,y) \text{ we have } y \Vdash \psi \\
x \Vdash \Diamond\psi & \text{iff there is a } y \in W \text{ such that } R(x,y) \text{ and } y \Vdash \psi
\end{array}
$$

It tells us that φ is necessarily true at x if φ is true in all worlds y accessible from x in a certain way. The meaning of R for each of the six readings of □ is shown in Table given below:

$\Box\phi$	$R(x,y)$
It is necessarily true that ϕ	y is possible world according to the information at x
It will always be true that ϕ	y is a future world of x
It ought to be that ϕ	y is an acceptable world according to the information at x
Agent Q believes that ϕ	y could be the actual world according to Q's beliefs at x
Agent Q knows that ϕ	y could be the actual world according to Q's knowledge at x
After any execution of P, ϕ holds	y is a possible resulting state after execution of P at x

A given binary relation R may be:

- *reflexive*: if, for every $x \in W$, we have $R(x,x)$;
- *symmetric*: if, for every $x, y \in W$, we have $R(x,y)$ implies $R(y,x)$;
- *serial*: if, for every x there is a y such that $R(x,y)$;
- *transitive*: if, for every $x, y, z \in W$, we have $R(x,y)$ and $R(y,z)$ imply $R(x,z)$;
- *Euclidean*: if, for every $x, y, z \in W$ with $R(x,y)$ and $R(x,z)$, we have $R(y,z)$;
- *functional*: if, for each x there is a unique y such that $R(x,y)$;
- *linear*: if, for every $x, y, z \in W$, we have that $R(x,y)$ and $R(x,z)$ together imply that $R(y,z)$, or y equals z, or $R(z,y)$;
- *total*: if for every $x, y \in W$ we have $R(x,y)$ or $R(y,x)$; and
- an *equivalence relation*: if it is reflexive, symmetric and transitive.

Correspondence Theory

Definition: A frame F =(W, R) is a set W of worlds and a binary relation R on W.

A frame is like a Kripke model, except that it has no labelling function. From any model we can extract a frame, by just forgetting about the labelling function as shown below: