Equal Sets:

Two or more set containing the same distinct element are said to be equal or identical.

Note: The order in which they are listed is not important.

Example 1:

A = {a, b, c, d}, B = {a, a, b, b, c, d}

A=B since we have four distinct elements.

Similarly,

If X = {1, 2, 3} and Y = {3, 1, 2} then X=Y

Example 2:

For P = {5, 10, 17, 26} and Q = {n^2 + 1: n is an integer, 1 < n < 6 }, then P = Q.

Cardinality of a set:

The number of element in set B is called the cardinality of B or the order of B, and is denoted by n(B).
If B = {a, b, c}, with three distinct elements, then the cardinality of the set B is n (B) = 3.

Finite and Infinite Set:

A set that has a finite number of elements is called a finite set.
Note: Empty set has no element but is finite.

Example 1:

A = {a, e, I, o, u} n (A) = finite

B = {1, 2, 3,….} n (B) = infinite

Power sets:

A set whose elements are themselves sets are said to be family or class of sets.
E.g. A {{3,5}, {6,9}, {1}} is a family of sets.

The power of set Y is the collection or class or family of all the subset of set Y. Any given set Y has $2^{()\,(Y)}$ subsets where () (Y) is the number of distinct element in set Y otherwise known as the order of the set Y.

Combination

Combination deals with possible sets from a given set where the order of a set is not important but the order of selection.

Combination deals with selection and choice of objects.

$^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

Exercise 13:

Solve $^5C_2$

Solution

$^5C_2 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!\,3!}$

$= \frac{5 \times 4 \times 3!}{2! \times 3!} = \frac{5 \times 4}{2 \times 1} = 10$

Exercise 14:

Prove that $^nC_r = {}^nC_{n-r}$

Solution

From the Left Hand Side

$^nC_r = \frac{n!}{r!(n-r)!}$

$= \frac{n!}{[n-(n-r)]!(n-r)!} = \frac{n!}{(n-r)!\,[n-(n-r)]!}$

$= {}^nC_{n-r}$

Random Variable

Variable means a letter, sign, shape e.t.c representing a value or liable to change.

Types of Random variable

1. Discrete random variable
2. Continuous random variable

Tip 2: You only equate to 1 only when you are looking for a constant.

Using the second property,

$\int_{-\infty}^{\infty} f(x)\, dx = 1$

$\int_{0}^{\infty} ke^{-3x}\, dx = 1$

$\left[\frac{e^{-3x}}{3}\right]_0^{\infty} = 1$

$-\left[\frac{e^{-3x}}{3}\right]_0^{\infty} = 1$

$-k\left[0 - \frac{1}{3}\right] = 1$

K =3

Substitute k into f(x)

f(x) = $3e^{-3x}$ for x > 0

I.

The interval (0.5 to 1) was clearly stated.

$= \int_{0.5}^{1} 3e^{-3x}\, dx$

Note: Don't equate to 1. Since, you are not looking for a constant.

$= 3\int_{0.5}^{1} e^{-3x}\, dx$

$= 3\left[\frac{e^{-3x}}{-3}\right]_{0.5}^{1} dx = -3\left[\frac{e^{-3x}}{3}\right]_{0.5}^{1}$

$= 3\left[e^{-3} - e^{-3(0.5)}\right] = -e^{-3} + e^{-1.5}$

= - 0.0498 + 0.2231 = 0.1733

DISTRIBUTION FUNCTION OR CUMULATIVE DISTRIBUTION FUNCTION (CDF)

Discrete probability is also known as probability mass function.

$C\left[\frac{x^{1/2}}{1/2}\right]_0^4 = 1$

$C\left[2x^{1/2}\right]_0^4 = 1$

$\left[2(4)^{1/2} - 2(0)^{1/2}\right] = 1$

$C[2(2)] = 1$

4C = 1

C = $1/4$

(ii)

$\int_0^x \frac{1/4}{\sqrt{x}} \; dx = 0$

$\int_0^x \frac{1}{4\sqrt{x}} \; dx = 0$

Tip 3: When ask to look for distribution function, the integral is always from 0 to x

$\frac{1}{4}\int_0^x \frac{1}{\sqrt{x}} \; dx = 0$

Tip (4): Because f(x) has x in it . We cannot substitute the UM(x) into it.

∴we change x in f(x) to t

$\frac{1}{4}\int_0^x \frac{1}{\sqrt{t}} dt = 0$

$= \left[\frac{t^{1/2}}{1/2}\right]_0^x$ dx

= $\frac{1}{4}\left[2t^{1/2}\right]_0^x$

= $\frac{1}{2}\left[t^{1/2}\right]_o^x$ dx

= $\left[\frac{\sqrt{t}}{2}\right]_0^x$ = $\left[\frac{\sqrt{x}}{2} - \frac{\sqrt{0}}{2}\right] = \frac{\sqrt{x}}{2}$

= $6m\left[\frac{x^2}{2} - \frac{x^2}{3}\right] - \left[\frac{x^2}{2} - \frac{x^2}{3}\right] = 0.5$

= $6m\left[\frac{1}{2} - \frac{1}{3}\right] - 0 = 0.5$

= $6m\left[\frac{3-2}{6}\right] = 0.5$

m(1) = 0.5

m = $\frac{1}{2}$

Exercise 20:

If the distribution function of the random variable X is given by f(x) = $1 - (1 + x)e^{-x}$ for x>0 and = 0 elsewhere. Find

(i) $P(X \leq 2)$
(ii) $P(1 < X < 3)$
(iii) $P(X > 4)$
(iv) (iv) The probability density function of X.

f(x) = $1 - (1 + x)e^{-x}$ for x>0

(i)

= $P(x \leq 2)$ (less than 2)

= $\left[1 - (1 + x)e^{-x}\right]_0^2$

= $[1 - (1 + x)e^{-x}]^2$ - $[1 - (1 + x)e^{-x}]^0$

= $[1 - 3e^{-2}]$ - $[1 - 1(1)]$

= $[1 - 3e^{-2} - 0]$

= $1 - 3e^{-2}$

(ii)

$m_x(t) = (t-1)^{-1}$............................(1)

= $\mu_1' = m_x\,(t=0) \implies \mu$

Differentiate eqn (1)

$= m_x' = (1-t)^{-2}$

= $(1-t)^{-2}$................................(2)

$\mu = m_x'\,(t=0)$

$\mu = (1-0)^{-2} = 1$

(iii)

Take the second derivative of eqn (2)

$m_x''\,(t) = 2(1-t)^{-3}$

$\mu_2'' = m_x''\,(t=0)$

$\mu_2'' = 2(1-0)^{-3} = 2(1)^{-3} = 2$

$\sigma^2 = \mu_2'' - \mu^2$

$\sigma^2 = 2(1)^2$

$\sigma^2 = 2 - 1 = 1$

Exercise 23:

Find the expected value of the random variable X whose probability density is given by f(x) = $\frac{1}{8}(x+1)$ for $2 < x < 4$, = 0 elsewhere.

Solution

E = $\int_{-\infty}^{\infty} x \cdot f(x)\,dx$

f(x) = $\frac{1}{8}(x+1)$

$= \int_2^4 x \cdot \frac{1}{8}\,(x+1)$

= $\frac{1}{8}\int_2^4 (x^2 + x)\,dx$

(iii)

V(x) = $\sigma^2 = \mu_2' - \mu^2$

Variance = V(x) = σ^2

Recall that;

$\mu = M_x'(t) = 2(2-t)^{-2}$

Differentiate

$\mu_2' = M_x''(t) = 4(2-t)^{-3}$

$\mu_2' = M_x''(t=0) = 4(2-0)^{-3}$

$\mu_2' = 4(2)^{-3}$

$\mu_2' = {}^{4}/_{8} = {}^{1}/_{2}$

$\sigma^2 = \mu_2' - \mu^2$

$\sigma^2 = {}^{1}/_{2} - \left({}^{1}/_{2}\right)^2$

$V(x) = \sigma^2 = {}^{1}/_{2} - {}^{1}/_{4}$

$V(x) = {}^{1}/_{4}$

Special Probability Distributions and Probability Densities

Exercise 30:

The average number of customers calling per minute on every Friday in a commercial bank in Lagos is 3.5. if the arrival time of customers can be modeled using Poisson distribution, find the probability that during one particular minute, there will be

(i) No customer calling

(ii) One customer calling

(iii) Three or more customer calling

2013/2014 PAST QUESTIONS AND ANSWERS

Solution:

f(x) = 6x (1 – x)

(i)

E(x) = $\int_0^1 x \cdot (6x - 6x^2)$

= $\int_0^1 6x^2 - 6x^3$

= $6\int_0^1 x^2 - x^3$

= $6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$

= $6\left[\left(\frac{1^3}{3} - \frac{1^4}{4}\right) - \left(\frac{0^3}{3} - \frac{0^4}{4}\right)\right]$

= $6\left[\left(\frac{1^3}{3} - \frac{1^4}{4}\right) - 0\right]$

E(x) = $6\left[\frac{4-3}{12}\right]$

E(x) = $\frac{6}{12} = \frac{1}{2}$

(ii)

$\sigma^2 = \mu_2' - \mu^2$ ……….eqn(1)

Let's solve for μ_2'

$\mu_2' = x^2 \cdot \int_0^1 6x - 6x^2$

= $\int_0^1 6x^3 - 6x^4$

= $6\int_0^1 x^3 - x^4$

= $6\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1$

= $6\left[\left(\frac{1^4}{4} - \frac{1^5}{5}\right) - \left(\frac{0^4}{4} - \frac{0^5}{5}\right)\right]$

= $6\left[\left(\frac{1^4}{4} - \frac{1^5}{5}\right) - 0\right]$

μ_2' = $6\left[\frac{5-4}{20}\right]$

μ_2' = E(x^2) = $\frac{6}{20} = \frac{3}{10}$

From eqn(1)

$\sigma^2 = \frac{3}{10} - (\frac{1}{2})^2$

= $\frac{3}{10} - \frac{1}{4}$

= $\frac{6-5}{20} = \frac{1}{20}$