Example: Find the range of T in the above example.

$T(1,0,0) = (1,0,-1)$

$T(0,1,0) = (-1,0,1)$

$T(0,0,1) = (0,1,0)$.

$\therefore R(T) = [(1,0,-1), (-1,0,1)(0,1,0)]$.

$= [(1,0,-1), (0,1,0)]$. [$\because$ $(-1,0,1)$ is a scalar multiple of $(1,0,-1)$ and can be dropped].

Let A be the standard matrix of T.

Then $T(x,y,z) = 0 \iff A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0_{3\times 3}$.

$\therefore$ $N(T)$ is the same as the nullspace of A.

$\therefore$ $\dim(\ker T) =$ nullity of T = nullity of A.

A [illegible] of $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} = [T(e_1), T(e_2), T(e_3)]$.

We know that $R(T)$ is the span of $T(e_1)$, $T(e_2)$ and $T(e_3)$.

$\therefore$ $R(T)$ is nothing but the column space of A.

$\therefore$ $\dim R(T) = \dim$ of col. space of $A = r(A)$.

We call this number the rank of T and denoted by $r(T)$

Thus we have

$$r(T) + n(T) = r(A) + n(A) = \text{Number of columns of } A$$

Rank-Nullity Theorem for linear transformations:

Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then

$$r(T) + n(T) = n = \text{dimension of the domain space}$$