$$R = \rho \frac{\ell}{A} \Rightarrow \rho = \frac{RA}{\ell} = \frac{\Omega . m^2}{m} = \Omega . m \quad \text{or} \quad \Omega . cm$$

$$A = \pi r^2 = \pi \left(\frac{d}{2} \right)^2$$

r = radians of section.

d = diameter of section.

Conductance (G):-

$$G = \frac{1}{R} = \frac{1}{\frac{\rho \ell}{A}} = \frac{A}{\rho \ell} \qquad \text{(Siemens (S))}$$

Prefix :-

pico	10^{-12}	P
nano	10^{-9}	n
micro	10^{-6}	µ
milli	10^{-3}	m
centi	10^{-2}	c
deci	10^{-1}	d
Kilo	10^{3}	K
Mega	10^{6}	M
Giga	10^{9}	G
Tera	10^{12}	T

Example: What is the resistance of 3 Km length of wire with cross section area 6 mm^2 and resistivity 1.8 µΩcm .

Solution:

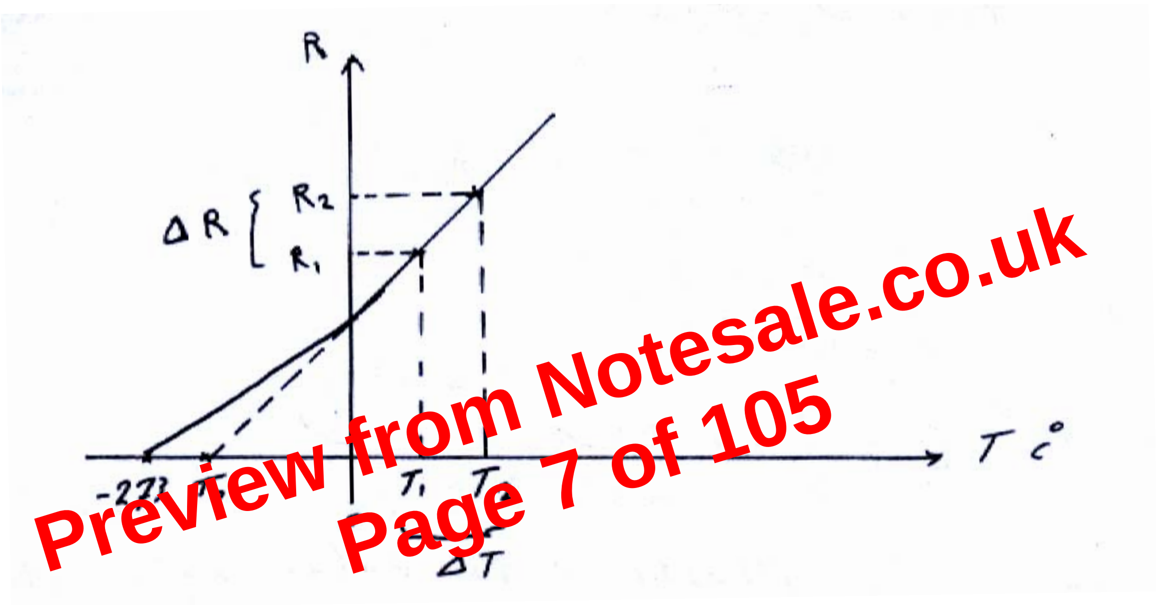
$$R = \frac{\rho \ell}{A} = \frac{\left(1.8 \times 10^{-6} \times 10^{-2}\right) \times \left(3 \times 10^{3}\right)}{\left(6 \times 10^{-6}\right)} = 9\Omega$$

Example: What is the resistance of 100 m length of copper wire with a diameter of (1 mm) and resistivity 0.0159 μΩm .

Solution:

$$R = \frac{\rho \ell}{A} = \frac{\left(0.0159 \times 10^{-6}\right) \times 100}{\left(\frac{d}{2}\right)^2 \pi} = \frac{\left(0.0159 \times 10^{-6}\right) \times 100}{\left(\frac{1 \times 10^{-3}}{2}\right)^2 \pi} = 2.02\ \Omega$$

<u>**Effect of Temperature on a resistance :-**</u>



$$\text{slop} = \frac{\Delta R}{\Delta T} = \text{constant} = \frac{R_2 - R_1}{T_2 - T_1} = \frac{R_2 - R}{T_2 - T} = \frac{R - R_1}{T - T_1}$$

Example: The resistance of material is 300 Ω at 10C°, and 400 Ω at 60C°. Find its resistance at 50 C° ?

Solution:

$$\text{slop} \ = \frac{R_2 - R_1}{T_2 - T_1} = \frac{400 - 300}{60 - 10} = 2\Omega / C^o$$

$$2 = \frac{R - R_1}{T - T_1} = \frac{R - 300}{50 - 10} = \frac{R - 300}{40}$$

$$R - 300 = 80 \Rightarrow R = 80 + 300 \Rightarrow R = 380\Omega$$

$E = I.[R_1 + R_2 + R_3] = I.R_T$

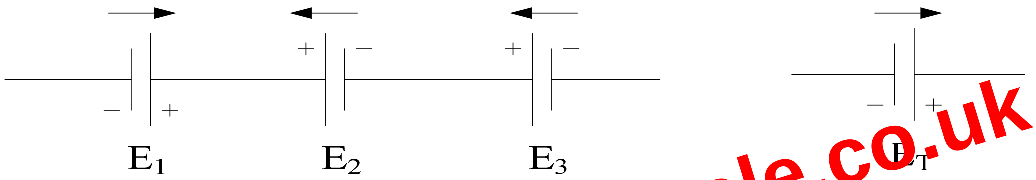
The current in the series circuit is the same through each series element &

$R_T = R_1 + R_2 + R_3 + \text{--------} + R_N$

$$I = \frac{E}{R_T} = \frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3}$$

$P_t = P_1 + P_2 + P_3 = E.I$

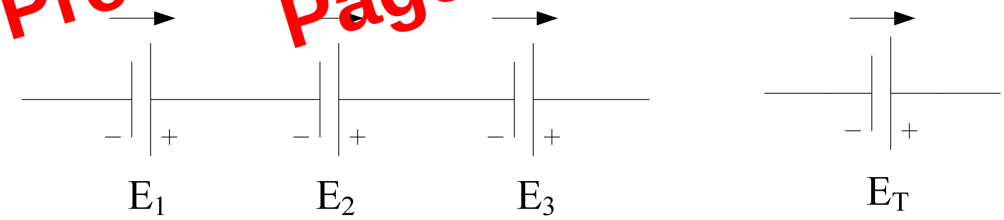
<u>Voltage Source in Series:-</u>



20V 10V 3V

$E_T = E_1 - E_2 - E_3$

$E_T = 20 - 10 - 3 = 7$

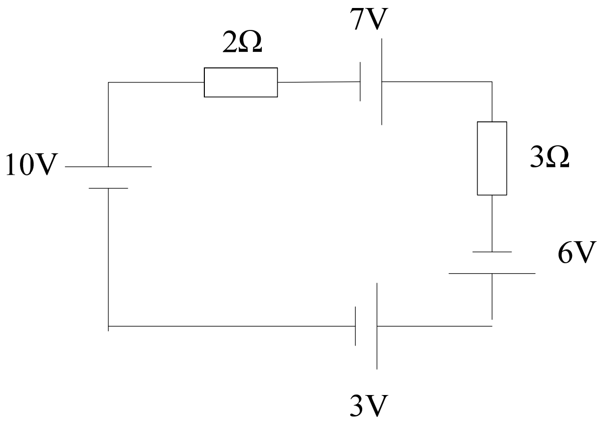


5V 6V 7V

$E_T = E_1 + E_2 + E_3$

$E_T = 5 + 6 + 7 = 18$

Example: Find the current for the following circuit diagram?



Solution:

$E_T = 10 + 7 + 6 - 3 = 20$ V

$R_T = 2 + 3 = 5$ Ω

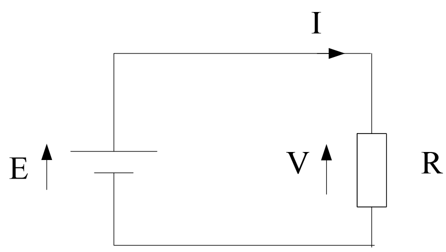
$$I = I_T = \frac{E_T}{R_T} = \frac{20}{5} = 4A$$

<u>Kirchoff's voltage law (K.V.L.):-</u>

The algebraic sum of all voltages around any closed path is zero.

$$\sum_{m=1}^{m} V_m = 0$$

Where m is the number of voltages in the path (loop) , and V_m is the m^{th} voltage .



$E - V = 0$

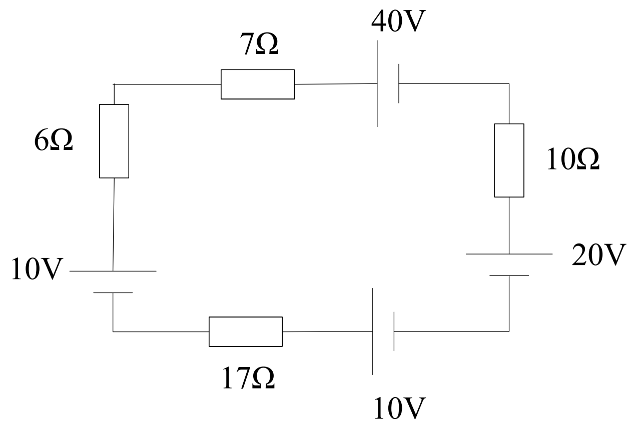
$E = V$

$$I = \frac{V}{R} = \frac{E}{R}$$

Example: For the following circuit diagram, Find I using:-

a) Ohm's law.

b) K.V.L.



Solution:

a) By applying ohm's law :-

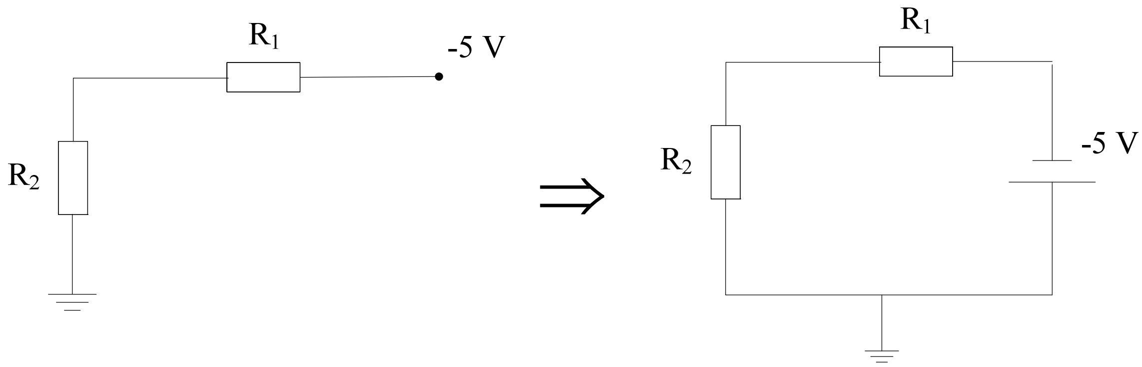
$$I = \frac{E_T}{R} = \frac{20+40-10-10}{10+7+6+17} = \frac{40}{40} = 1A$$

b) By applying K.V.L. :-

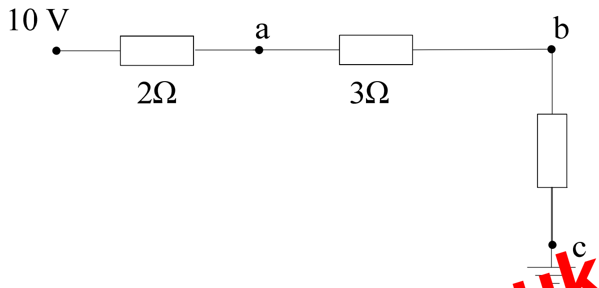
$10 + 6I + 7I - 40 + 10I - 20 + 10 + 17I = 0$

$10 - 40 - 20 + 10 + I(6 + 7 + 10 + 17) = 0$

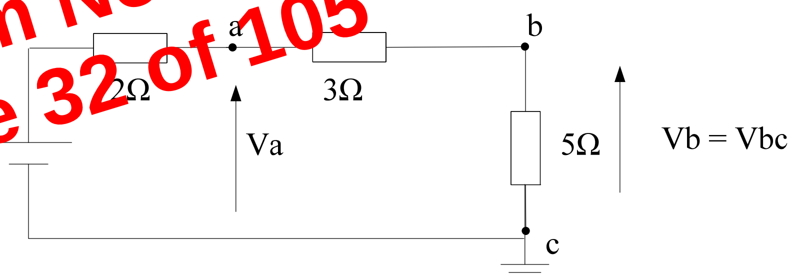
$-40 = -I(40) \Rightarrow I = \frac{40}{40} = 1A$



Example :- Find V_a , V_b , V_c , V_{ab} , V_{ac} and V_{bc} for the following diagram .



Solution :-



$R_T = R_1 + R_2 + R_3$

$= 2 + 5 + 3 = 10\ \Omega$

$$I = \frac{E}{R_T} = \frac{10}{10} = 1A$$

$E - V_2 - V_a = 0$

$V_a = E - V_2 = 10 - (2 * 1) = 8\ V$

$V_b = V_5 = (1 * 5) = 5\ V = V_{bc}$; $V_c = 0\ V$

or $E - V_2 - V_3 - V_b = 0$ $\Rightarrow V_b = E - V_2 - V_3 = 10 - 2 - 3 = 5\ V$

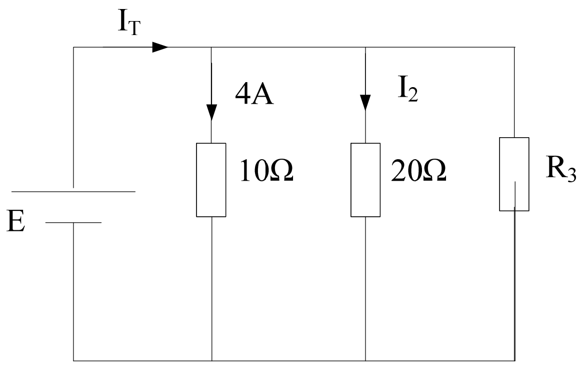
$V_{ab} = V_a - V_b = 8 - 5 = 3\ V$

$V_{ac} = V_a - V_c = 8 - 0 = 8\ V$

$V_{bc} = V_b - V_c = 5 - 0 = 5\ V$

Example :- For the parallel network in fig. below , find :-

a) R_3 , b) E , c) I_T , I_2 , d) P_2 ; given that $R_T = 4\ \Omega$?



Solution :-

a)

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{4} = \frac{1}{10} + \frac{1}{20} + \frac{1}{R_3}$$

$$0.25 = 0.1 + 0.05 + \frac{1}{R_3}$$

$$0.25 - 0.1 - 0.05 = \frac{1}{R_3}$$

$$0.1 = \frac{1}{R_3} \Rightarrow R_3 = \frac{1}{0.1} = 10\Omega$$

b) $E = V_1 = I_1 R_1 = 4 * 10 = 40\ V$

c)

$$I_T = \frac{E}{R_T} = \frac{40}{4} = 10A$$

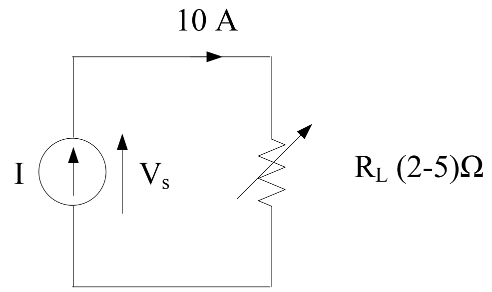
$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{40}{20} = 2A$$

d) $P_2 = I_2^2 R_2 = (2)^2.(20) = 80W$

or $P_2 = \dfrac{V_2^2}{R_2}$, or $P_2 = I_2 V_2$

Current Source :-

Example :- Find the voltage (V_s) for the circuit below:

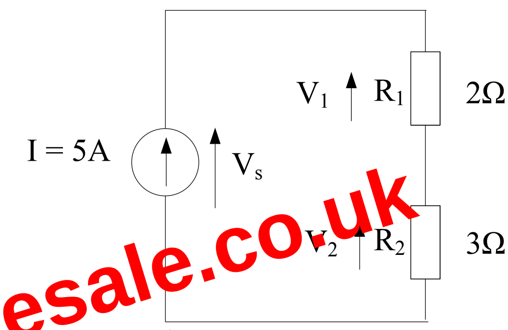


Solution :-

$V_s = IR_L = 10 * 2 = 20$ V if $R_L = 2\ \Omega$

$V_s = IR_L = 10 * 5 = 50$ V if $R_L = 5\ \Omega$

Example :- Calculate V_1 , V_2 , V_s for the following cct.:



Solution :-

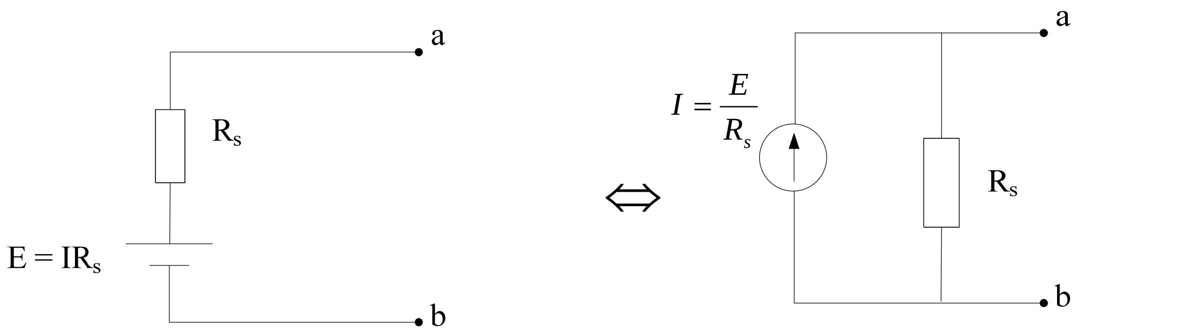
$V_1 = IR_1 = 5 * 2 = 20$ V

$V_2 = IR_2 = 5 * 3 = 15$ V

$V_s = V_1 + V_2 = 10 + 15 = 25$ V

Source Conversion :-

A voltage source with voltage E and series resistor R_s can be replaced by a current source with a current I and parallel resistor R_s as shown :-



Current source to voltage source

Voltage source to current source

Solution :-

$$D = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 0 \\ 0 & 4 & 2 \end{bmatrix} \begin{matrix} 1 & 2 \\ -2 & 1 \\ 0 & 4 \end{matrix}$$

$$\therefore D = [1*1*2 + 2*0*0 + 3*(-2)*4] - [0*1*3 + 4*0*1 + 2*-2*2]$$

$$D = [2 + 0 - 24] - [0 + 0 + 8] = -22 + 8 = -14$$

Example :- Find V_1 , V_2 , V_3 from the following equations :-

$2V_1 + 4V_2 + 2V_3 = 8$

$5V_1 - 2V_2 - 10V_3 = 18$

$V_1 + 8V_2 - 20V_3 = -8$

Solution :-

$$\begin{bmatrix} 2 & 4 & 2 \\ 5 & -2 & -10 \\ 1 & 8 & -20 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 18 \\ -8 \end{bmatrix}$$

$$V_1 = \frac{D_1}{D} = \frac{\begin{bmatrix} 8 & 4 & 2 \\ 18 & -2 & -10 \\ -8 & 8 & -20 \end{bmatrix} \begin{matrix} 8 & 4 \\ 18 & -2 \\ -8 & 8 \end{matrix}}{\begin{bmatrix} 2 & 4 & 2 \\ 5 & -2 & -10 \\ 1 & 8 & -20 \end{bmatrix} \begin{matrix} 2 & 4 \\ 5 & -2 \\ 1 & 8 \end{matrix}}$$

$$V_1 = \frac{[8*(-2)*(-20) + 4*(-10)*(-8) + 2*18*8] - [(-8)*(-2)*2 + 8*(-10)*8 + (-20)*18*4]}{[2*(-2)*(-20) + 4*(-10)*1 + 2*5*8] - [1*(-2)*2 + 8*(-10)*2 + (-20)*5*4]}$$

$$V_1 = \frac{2976}{684} = 4.35V$$

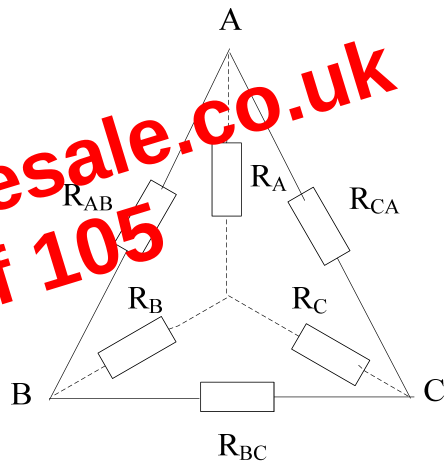
$$V_2 = \frac{D_2}{D} = \frac{\left[\begin{array}{ccc} 2 & 8 & 2 \\ 5 & 18 & -10 \\ 1 & -8 & -20 \end{array}\right] \begin{array}{cc} 2 & 8 \\ 5 & 18 \\ 1 & -8 \end{array}}{684}$$

$$V_3 = \frac{D_3}{D} = \frac{\left[\begin{array}{ccc} 2 & 4 & 8 \\ 5 & -2 & 18 \\ 1 & 8 & -8 \end{array}\right] \begin{array}{cc} 2 & 4 \\ 5 & -2 \\ 1 & 8 \end{array}}{684}$$

Star – Delta ($Y \rightarrow \Delta$) and Delta – Star ($\Delta \rightarrow Y$) transformation :-

1.) Delta – Star ($\Delta \rightarrow Y$) transformation :-

If the value of R_{AB} , R_{CA} , R_{BC} are known, and we need to get the values of R_A , R_B , R_C ; then :-



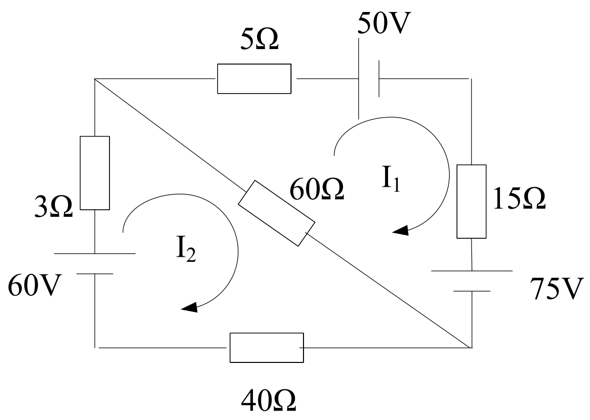
$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{CA} + R_{BC}}$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{CA} + R_{BC}}$$

$$R_C = \frac{R_{CA} R_{BC}}{R_{AB} + R_{CA} + R_{BC}}$$

If $R_{Ab} = R_{BC} = R_{CA} = R_{\Delta}$, in this case $R_A = R_B = R_C = \frac{R_{\Delta}}{3} = R_Y$

or $R_Y = \frac{R_{\Delta}}{3}$



$$-I_1 \, (5+15+60) + 60I_2 - 50 - 75 = 0$$

$$-I_2 \, (3+60+40) + 60I_1 + 60 = 0$$

Rearrange:-

$$-80I_1 + 60I_2 = 125 \qquad \text{-------------------} \qquad (1)$$

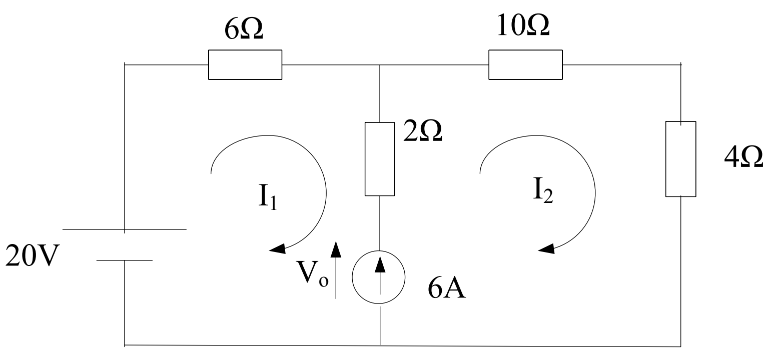
$$60I_1 - 103I_2 = -60 \qquad \text{-------------------} \qquad (2)$$

$$I_1 = \frac{D_1}{D}$$

اكمل الحل

$$I_2 = \frac{D_2}{D}$$

Example(6):- Solve the following circuit diagram:



Kcl at C:

$$I_6 + I_1 - I_4 = 0$$

$$(V_B - V_C) G_6 + [(V_A - V_C)- E_1] G_1 - V_C G_4 = 0$$

Rearrange:

A : $(V_B - V_A) G_5 - V_A G_3 + (V_C - V_A) G_1 - E_1G_1 = 0$

B : $(V_A - V_B) G_5 + (V_C - V_B) G_6 - V_B G_2 + E_2 G_2 = 0$

C : $(V_B - V_C) G_6 - V_C G_4 + (V_A - V_C) G_1 - E_1G_1 = 0$

Hence, we can arrange the above equations in the following form:-

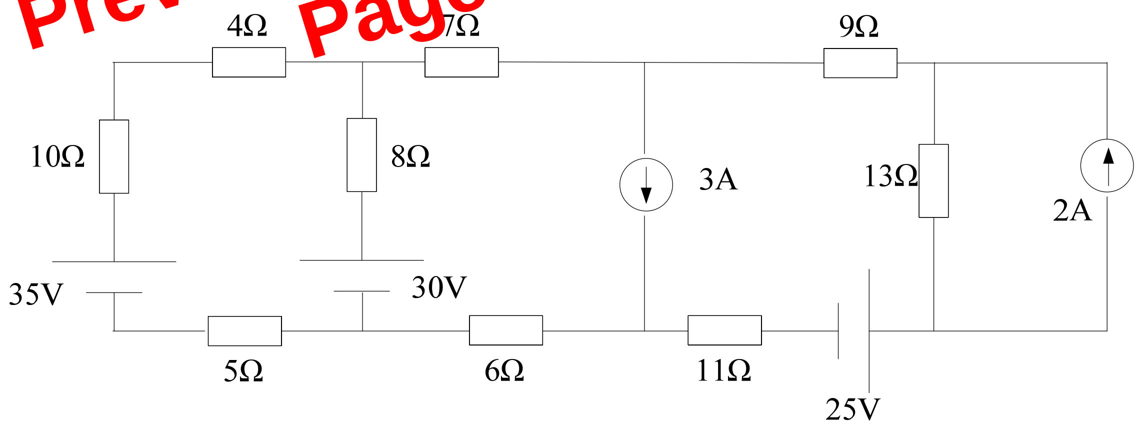
A : $- V_A (G_1 + G_3 + G_5) + V_BG_5 + V_CG_1 + E_1G_1 = 0$ ----------- (1)

B : $- V_B (G_2 + G_5 + G_6) + V_AG_5 + V_CG_6 + E_2G_2 = 0$ ----------- (2)

C : $- V_C (G_1 + G_4 + G_6) + V_AG_1 + V_BG_6 - E_1G_1 = 0$ ----------- (3)

Then, we can find V_A , V_B , V_C by the determinate method .

Example 2 :- Solve the following circuit diagram using nodal voltage .



Solution:

First we simplify the circuit and make a less nodal point.

المحاضرة (9):

د. سمير عبد الخالق عزيز

د. غيداء كائن صالح

قسم الهندسة الكهروميكانيكية/ المرحلة الاولى

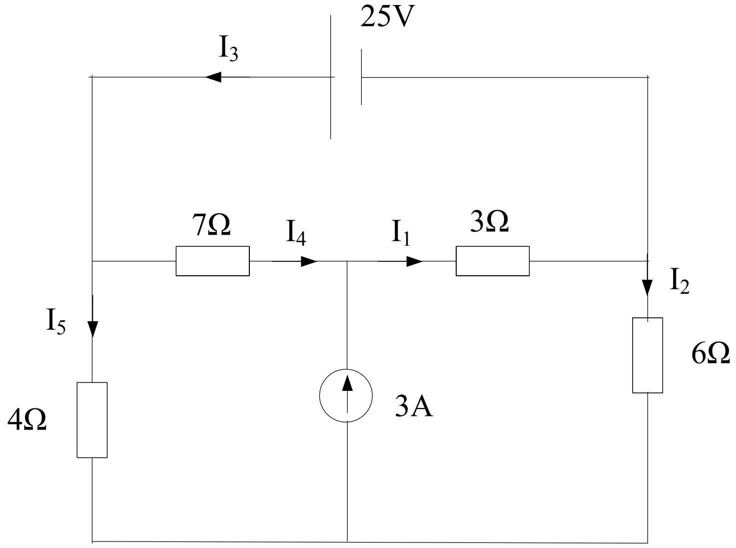
Network Theorems:-

1- Superposition Theorem:-

In any circuit network contain more than one sources (voltage or current) to find the current (or voltage) in a certain part of a network , remove the sources of the network and find the current (or voltage) in the existence of only one source each time. The resultant current (or voltage) will be the algebraic sum of current (or voltage) due to all sources when acting independently once a time .

(Removing the sources means:- Short circuiting the voltage source and open circuiting the current source**)** .

Example 1:- In the following circuit diagram, find all branch current's using superposition theorem:



المحاضرة (10):

د. سمير عبد الخالق عزيز د. غيداء كائن صالح

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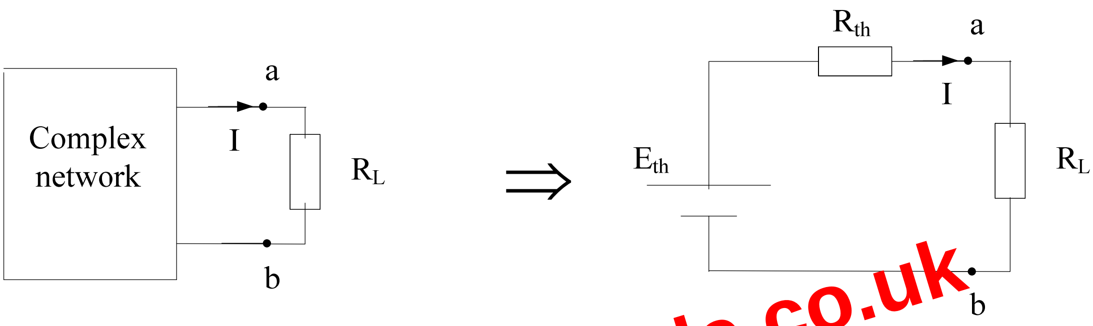
2-) Thevenin's Theorems:-

تستخدم في اغلب الاحيان اذا كان المطلوب ايجاد التيار او الفولتية في مقاومة محددة في الدائرة .

Any two terminal linear network can be replaced by an equivalent circuit of a voltage source (E_{th}) and a series resistor (R_{th}); as shown in figure below:-

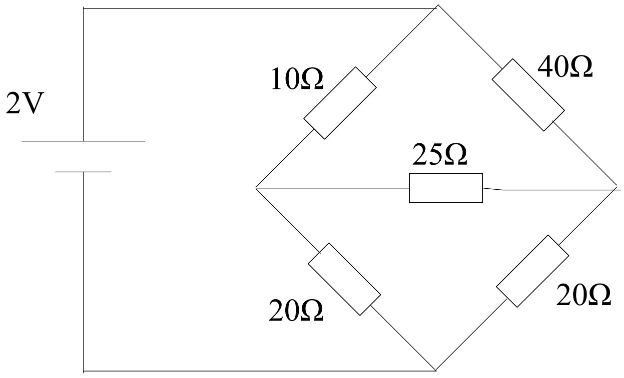


Hence; $I = \frac{E_{th}}{R_{th} + R_L}$

Steps to find E_{th} & R_{th} :-

1. Remove that portion of the network across which the Thevenins equivalent circuit is to be find.
2. Mark the terminals of the remaining two – terminal network.
3. Calculate R_{th} by first setting all sources to zero (voltage sources are replaced by short circuits and current sources are replaced by open circuit), and finding the resultant resistance between the two marked terminals.
4. Calculate E_{th} by first returning all sources to their origin positions and finding the open circuit voltage between the marked terminals.
5. Draw the Thevenins equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Example 2:- Find the current in the 25Ω resistor for the following circuit network?

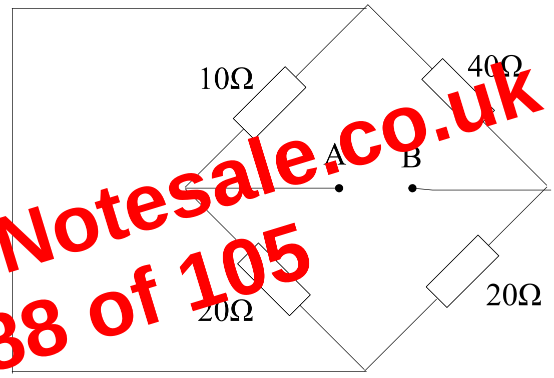


Solution:-

1.) Find R_{th} :

$$R_{th} = (10 // 20) + (40 // 20)$$

$$\therefore R_{th} = \frac{10*20}{10+20} + \frac{40*20}{40+20} = 20\Omega$$

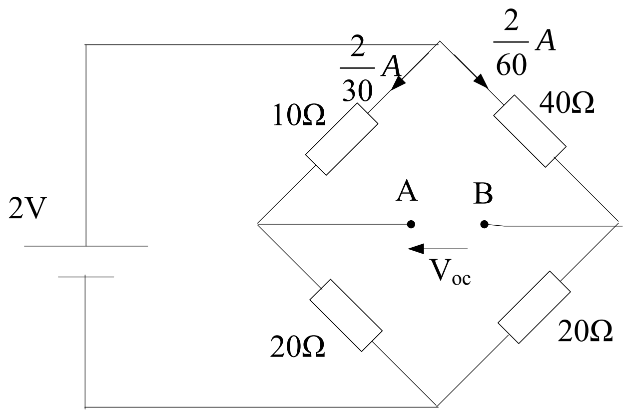


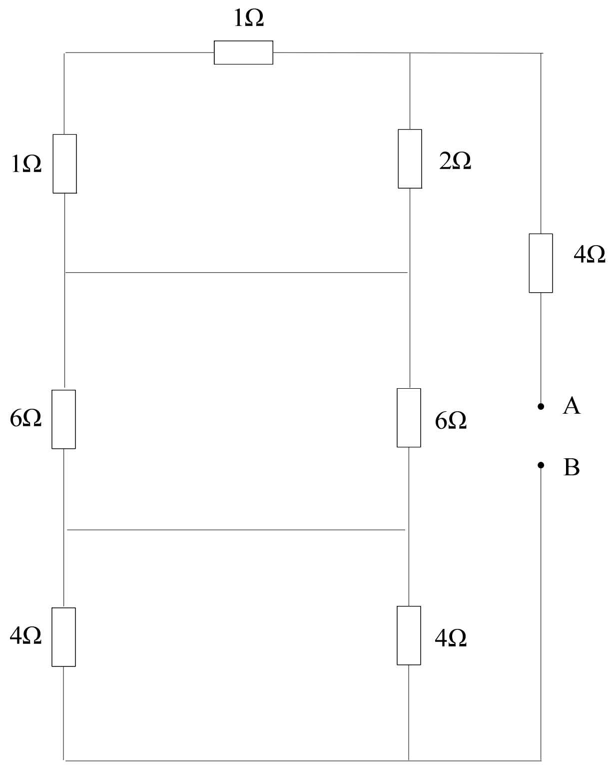
2.) Find E_{th} :

$$V_{oc} + \left(10*\frac{2}{30}\right) - \left(40*\frac{2}{60}\right) = 0$$

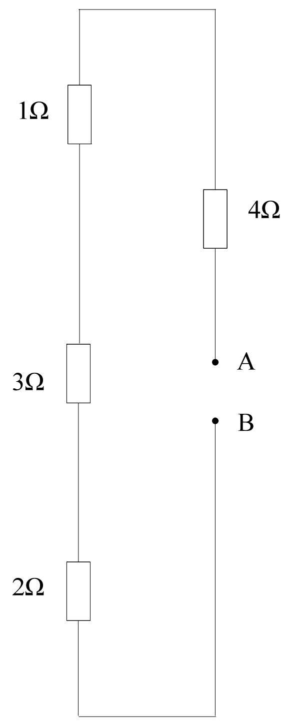
$$V_{oc} = \frac{80}{60} - \frac{20}{30} = \frac{40}{60} = 0.67V$$

$$\therefore E_{th} = 0.67V$$



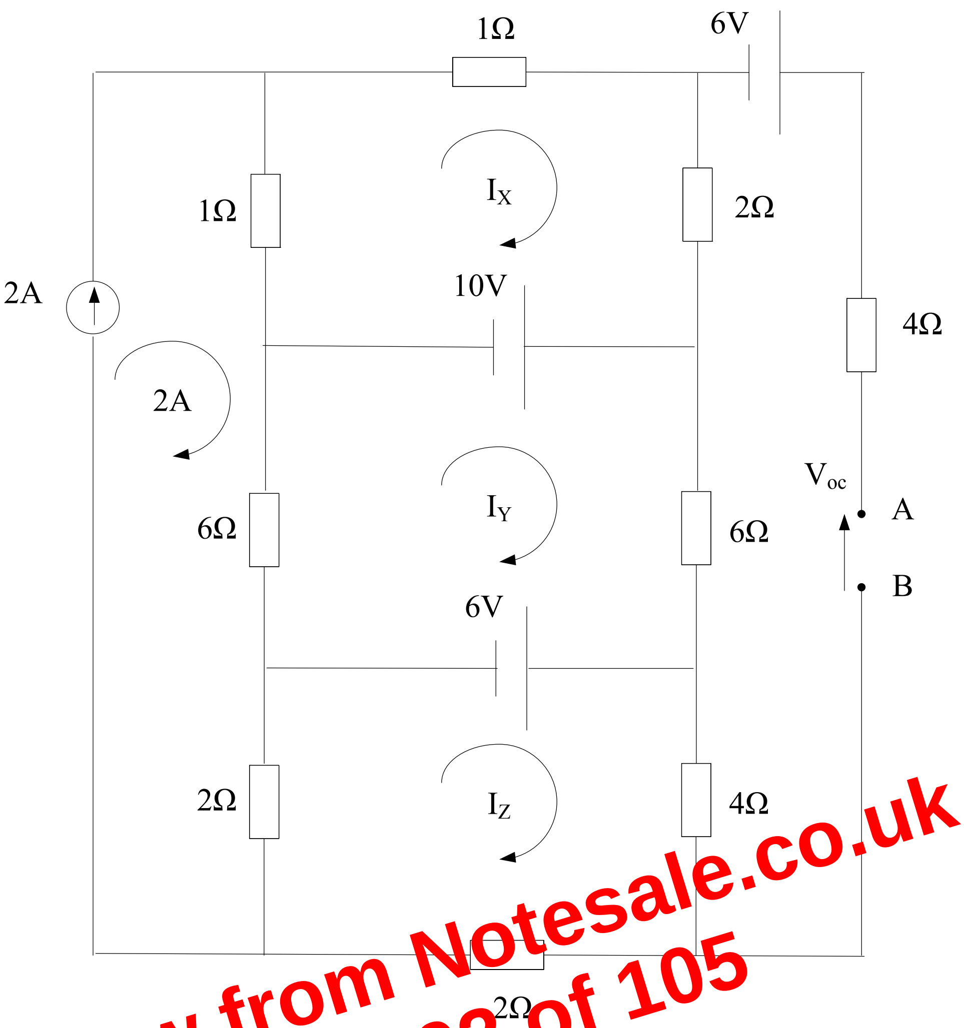


⇒



$$\therefore R_{eq.} = R_{th} = 4 + 1 + 3 + 2 = 10\Omega = R$$

2.) <u>Find E_{th} :-</u>



$-4I_x - 10 + (1*2) = 0 \Rightarrow I_x = -2A$

$-12I_y + 10 - 6 + (2*6) = 0 \Rightarrow I_y = 1.33A$

$-8I_z + 6 + (2*2) = 0 \Rightarrow I_z = 1.25A$

From KVL

$\therefore 6 - V_{oc} + (4*1.25) + (6*1.33) - (2*2) = 0$

$V_{oc} = 6 + 5 + 8 - 4 = 15V$

$$I = \frac{15}{10+10} = 0.75A$$