Design an analog Butterworth filter that has a -2 dB *pass band attenuation at a frequency of* 20 rad/sec *an atleast* -10 dB *stopband attenuation at* 30 rad/sec.

Given $\alpha_p = 2$ dB, $\Omega_p = 20$ rad/sec

$\alpha_s = 10$ dB, $\Omega_s = 30$ rad/sec

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\,\alpha_s} - 1}{10^{0.1\,\alpha_p} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

$$\geq \frac{\log \sqrt{\dfrac{10 - 1}{10^{0.2} - 1}}}{\log \dfrac{30}{20}}$$

≥ 3.37 Rounding off N to the next highest integer we get

$$N = 4$$

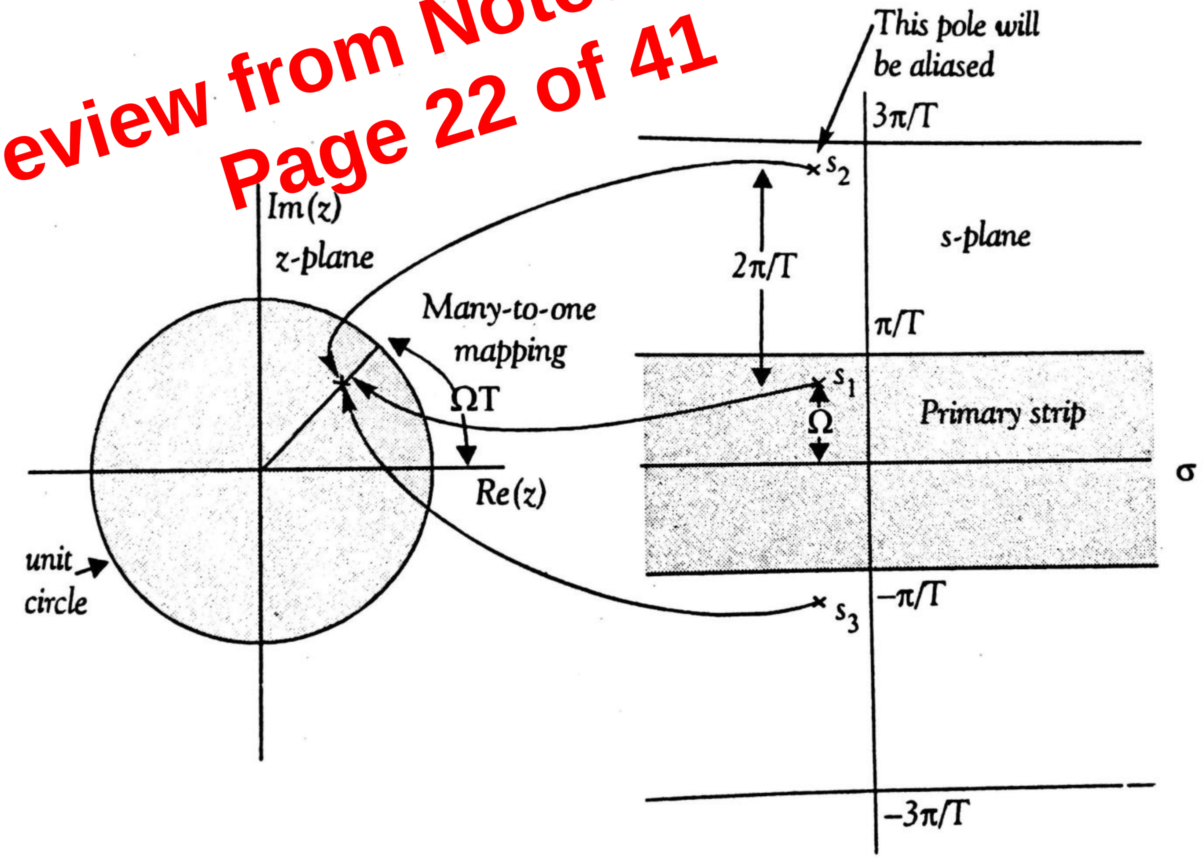

Fig. 5.18 Impulse invariant pole mapping

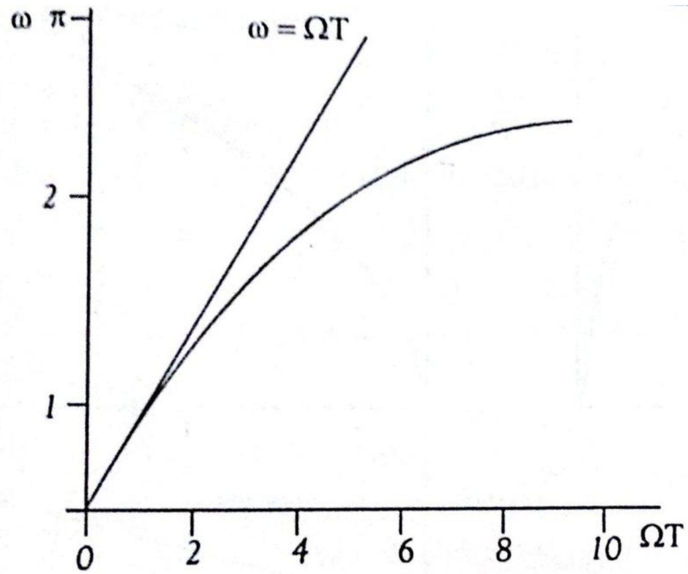

Fig. 5.19 Relationship between Ω and ω.

Apply bilinear transformation to $H(s)=\dfrac{2}{(s+1)(s+2)}$ *with* $T=1$ *sec and find* $H(z)$

Given $H(s)=\dfrac{2}{(s+1)(s+2)}$

Substitute $s=\dfrac{2}{T}\left[\dfrac{1-z^{-1}}{1+z^{-1}}\right]$ in $H(s)$ to get $H(z)$

$$H(z)=H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$=\frac{2}{(s+1)(s+2)}\Bigg|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

Design a third order Butterworth digital filter using impulse invariant technique. Assume sampling period $T = 1$ sec.

From the table 5.1, for $N = 3$, the transfer function of a normalised Butterworth filter is given by

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$= \frac{A}{s+1} + \frac{B}{s+0.5+j0.866} + \frac{C}{s+0.5-j0.866}$$

First we design a lowpass filter for the given specifications and use suitable transformation to obtain transfer function of highpass filter.

The order of the filter

$$N = \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}} = \frac{\log \sqrt{\dfrac{10^{0.1(10)} - 1}{10^{0.1(3)} - 1}}}{\log \dfrac{7265}{2235}} = \frac{\log 3}{\log 3.25} = \frac{0.4771}{0.5118} = 0.932$$

Therefore, we take $N = 1$.

The first-order Butterworth filter for $\Omega_c = 1$ rad/sec is $H(s) = \dfrac{1}{1+s}$