An equivalent version of Theorem 1.2 was probably first recorded as Proposition 14 of Book IX in Euclid's Elements, before it came to be known as the Fundamental Theorem of Arithmetic. However, the first correct proof was given by Carl Friedrich Gauss in his *Disquisitiones Arithmeticae*.

Carl Friedrich Gauss is often referred to as the 'Prince of Mathematicians' and is considered one of the three greatest mathematicians of all time, along with Archimedes and Newton. He has made fundamental contributions to both mathematics and science.

**Carl Friedrich Gauss (1777 – 1855)**



The Fundamental Theorem of Arithmetic says that every composite number can be factorised as a product of primes. Actually it says more. It says that given any composite number it can be factorised as a product of prime numbers in a **'unique'** way, except for the order in which the primes occur. That is, given any composite number there is one and only one way to write it as a product of primes, as long as we are not particular about the order in which the primes occur. So, for example, we regard $2 \times 3 \times 5 \times 7$ as the same as $3 \times 5 \times 7 \times 2$, or any other possible order in which these primes are written. This fact is also stated in the following form:

*The prime factorisation of a natural number is unique, except for the order of its factors.*

In general, given a composite number $x$, we factorise it as $x = p_1 p_2 \ldots p_n$, where $p_1, p_2, \ldots, p_n$ are primes and written in ascending order, i.e., $p_1 \leq p_2 \leq \ldots \leq p_n$. If we combine the same primes, we will get powers of primes. For example,

$$32760 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 13 = 2^3 \times 3^2 \times 5 \times 7 \times 13$$

Once we have decided that the order will be ascending, then the way the number is factorised, is unique.

The Fundamental Theorem of Arithmetic has many applications, both within mathematics and in other fields. Let us look at some examples.

**Example 5 :** Consider the numbers $4^n$, where $n$ is a natural number. Check whether there is any value of $n$ for which $4^n$ ends with the digit zero.

**Solution :** If the number $4^n$, for any $n$, were to end with the digit zero, then it would be divisible by 5. That is, the prime factorisation of $4^n$ would contain the prime 5. This is

So, HCF (6, 72, 120) = $2^1 \times 3^1 = 2 \times 3 = 6$

$2^3$, $3^2$ and $5^1$ are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the three numbers.

So, LCM (6, 72, 120) = $2^3 \times 3^2 \times 5^1 = 360$

**Remark :** Notice, $6 \times 72 \times 120 \neq$ HCF (6, 72, 120) $\times$ LCM (6, 72, 120). So, the product of three numbers is not equal to the product of their HCF and LCM.

## EXERCISE 1.2

1. Express each number as a product of its prime factors:
   (i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429
2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.
   (i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54
3. Find the LCM and HCF of the following integers by applying the prime factorisation method.
   (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25
4. Given that HCF (306, 657) = 9, find LCM (306, 657).
5. Check whether $6^n$ can end with the digit 0 for any natural number $n$.
6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.
7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

## 1.4 Revisiting Irrational Numbers

In Class IX, you were introduced to irrational numbers and many of their properties. You studied about their existence and how the rationals and the irrationals together made up the real numbers. You even studied how to locate irrationals on the number line. However, we did not prove that they were irrationals. In this section, we will prove that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and, in general, $\sqrt{p}$ is irrational, where $p$ is a prime. One of the theorems, we use in our proof, is the Fundamental Theorem of Arithmetic.

Recall, a number '$s$' is called *irrational* if it cannot be written in the form $\frac{p}{q}$, where $p$ and $q$ are integers and $q \neq 0$. Some examples of irrational numbers, with

## A NOTE TO THE READER

You have seen that :

HCF $(p, q, r) \times$ LCM $(p, q, r) \neq p \times q \times r$, where $p, q, r$ are positive integers (see Example 8). However, the following results hold good for three numbers $p, q$ and $r$ :

$$\text{LCM}(p, q, r) = \frac{p \cdot q \cdot r \cdot \text{HCF}(p, q, r)}{\text{HCF}(p, q) \cdot \text{HCF}(q, r) \cdot \text{HCF}(p, r)}$$

$$\text{HCF}(p, q, r) = \frac{p \cdot q \cdot r \cdot \text{LCM}(p, q, r)}{\text{LCM}(p, q) \cdot \text{LCM}(q, r) \cdot \text{LCM}(p, r)}$$