

flow of charge through a certain section. Its unit is ampere. A current is said to be of one ampere when a charge of 1 coulomb flows through a section per second.

Mathematically,

$$i = \frac{dq}{dt} \quad \dots(1.1)$$

If charge q is expressed in coulomb and time in second, 1 amp flow of current through a section is equivalent to approx. flow of 6.24×10^{18} electrons per second through the section.

Law of conservation of energy states that energy can neither be created nor destroyed. However, its form can be converted i.e. energy can be converted from one form of energy to another e.g. Electro-mechanical energy conversion, Electro-chemical energy conversion, MHD energy conversion, photoelectric energy conversion etc. In all these cases the function of each of these sources of electric energy is the same in terms of energy and charge i.e. the energy is spent as work for transporting charge from one point to another in a circuit. The movement of charges contribute to current and the amount of work done per unit charge is the potential difference between the two points. The electronic charge flows from a lower potential to a higher potential and these contribute to electronic current, whereas the conventional current is considered to flow from higher potential to lower potential.

If a differential charge dq is given a differential energy dw , the rise in potential of the charge

$$v = \frac{dw}{dq} \quad \dots(1.2)$$

If potential is multiplied by the current $\frac{dq}{dt}$

$$v \times i = \frac{dw}{dq} \times \frac{dq}{dt} = p \quad \dots(1.3)$$

which gives rate of change of energy with time and is equal to power. Thus,

$$\text{power } p = v \times i$$

$$\text{Since } p = \frac{dw}{dt}$$

In general

$$dw = p dt$$

$$W = \int vidt \quad \dots(1.4)$$

1.2 CIRCUIT ELEMENTS

Basically there are three circuit elements in electric circuits. These are resistance, inductance and capacitance. We shall study these elements in terms of the physical phenomenon, occurring in these elements, the field phenomenon associated with them and finally its relevance in circuit theory.

Resistance

When electrons flow through a material, they collide with other electrons and atomic particles. These collisions being inelastic, these electrons lose energy. This loss of energy per

Here C is known as capacitance between the parallel plates separated by a distance d . So, capacitance depends upon the geometry of the conductors and ϵ . If q is measured in coulombs and V in volts, C is measured in Farads. If q_0 is the initial charge on the plates of the capacitor and while charging if the charge increases linearly with time, the charge at any instant is given as

$$q = q_0 + Kt$$

Hence,

$$i = \frac{dq}{dt} = K$$

or
$$q = q_0 + \int_0^t i dt \quad \dots(1.20)$$

Also we know that $q = C V$

Assuming C to be constant, it is seen that, an instantaneous change in q would mean an instantaneous change in voltage V . But from the integral equation for charge q , it is found that for finite value of i , the charge can't change at $t = 0$ and it remains q_0 . Therefore, the voltage across a capacitor can't change instantaneously.

Let us take up a few examples to illustrate the application of some of the relations obtained in the previous discussions.

Example 1.1: The current waveform in a series circuit of 500Ω resistance and a 500 mH inductor is given in Fig. E.1.1 (a). Determine the voltage wave form in the resistor, in the inductor and the total voltage.

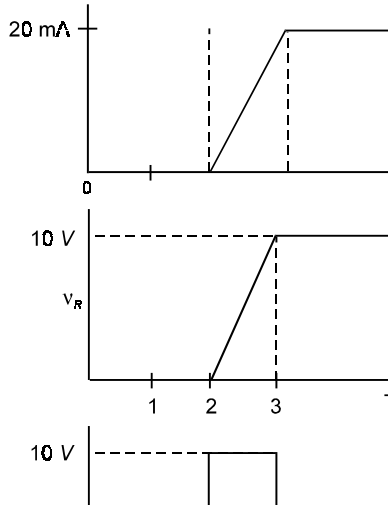


Fig. E.1.1

From 4 to 6 m sec, $v = 0$ since there is no change in the current it continues to be same i.e. 100 mA

From 6 to 10 m sec.

$$i = \frac{1}{0.1} \int_6^{10} (-5) dt = -200 \text{ mA}$$

It decreases linearly between 6 to 10 m sec. Hence the current at 10 m sec. is $-200 + 100 = -100 \text{ mA}$

From 10 to 12 m sec. since voltage is zero there is no change in current and it is -100 mA , from 12 to 13 m sec, the voltage is 10 volts. Hence current increases linearly by an amount

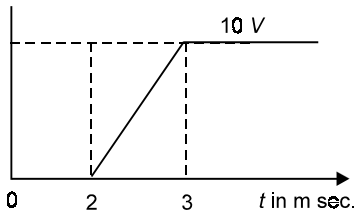
$$\frac{1}{0.1} \int_{12}^{13} 10 dt = 100 \text{ mA}$$

Beyond 13 m sec the current is zero.

Let us next consider application of non-sinusoidal voltages and currents in a capacitive circuit.

Example 1.3: Consider the application of a voltage wave form shown in Fig. E1.3 (a) to a $0.2 \mu\text{F}$ capacitor.

Determine the current wave form.



(a) Voltage.

(b) Current wave form.

Fig. E.1.3

Solution: Since current in a capacitor is given by

$$i = C \frac{dv}{dt}$$

From 0 to 2 m sec $dv = 0$ hence $\frac{0}{2} = 0$ Hence the current in this period is zero.

From 2 to 3 m sec the voltage increases linearly from 0 to 10 volts in 1 m sec. Hence

$$v = kt, k = \frac{10}{1 \times 10^{-3}} = 10 \times 10^3 \text{ v/sec or } 10 \text{ V/m sec}$$

$$\begin{aligned} i &= C \frac{dv}{dt} = 0.2 \times 10^{-6} \times k \\ &= 0.2 \times 10^{-6} \times 10 \times 10^3 \\ &= 2 \text{ mA} \end{aligned}$$

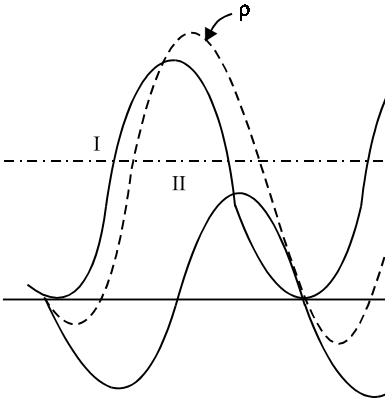


Fig. 1.6 Active, reactive and total power in a single phase circuit.

- (i) The component P marked I pulsates around the same average power $VI \cos \phi$ but never goes negative as the factor $(1 - \cos 2\omega t)$ can at the most be zero but it will never go negative. We define this average power as the real power P which physically means the useful power being consumed.
- (ii) The component marked II contains the term $\sin \phi$ which is negative for capacitive circuit and is positive for inductive circuit. This component pulsates and has zero as its average value. This component is known as reactive power as it travels back and forth on the line without doing any useful work.

Equation (1.23) is rewritten as

$$p = P (1 - \cos 2 \omega t) - Q \sin 2 \omega t \quad \dots(1.24)$$

Both P and Q have the same dimensions of watts but to emphasise the fact that Q represents a nonactive power, it is measured in terms of voltamperes reactive i.e. VAr .

The term Q requires more attention because of the interesting property of $\sin \phi$ which is $-ve$ for capacitive circuits and is $+ve$ for inductive circuits. This means a capacitor is a generator of positive reactive VAr , a concept which is usually adopted by power system engineers. So it is better to consider a capacitor supplying a lagging current rather than taking a leading current (Fig. 1.7).

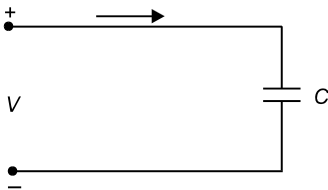


Fig. 1.7 $V-I$ relations in a capacitor.

Consider a circuit in which an inductive load is shunted by a capacitor. If Q is the total reactive power requirement of the load and Q' is the reactive power that the capacitor can generate, the net reactive power to be transmitted over the line will be $(Q - Q')$. This is the basic concept of synchronous phase modifiers for controlling the voltage of the system. The phase modifier controls the flow of reactive power by suitable excitation and hence the

is a non-reciprocal network. R, L, C , are in general linear, passive, bilateral, time invariant and lumped elements. Since L and C can store energy in the form of electro-magnetic and electro-static fields respectively their representation needs special mention. In general voltage across capacitor is given by

$$v(t) = \frac{1}{C} \int i(t) dt + v(o) \quad \dots(1.48)$$

where $v(o)$ indicates the voltage due to initial charge. The capacitor which has initial voltage $v(o)$ can be represented as shown in Fig. 1.13(a).

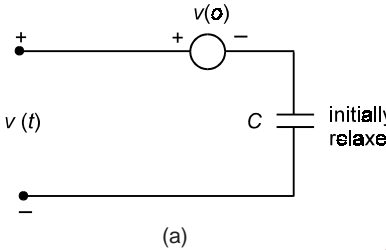


Fig. 1.13 Initial condition for a capacitor and inductor.

Similarly if an inductor initially has a current $i(o)$ the current at anytime t is given by

$$i(t) = \frac{1}{L} \int v(t) dt + i(o) \quad \dots(1.49)$$

Therefore, $i(o)$ could be considered as a d.c. current source in parallel with an initially relaxed (initial condition zero) inductor as shown in Fig 1.13(b).

1.4 THE TRANSFORMER

It is a two port network. An ideal transformer shown in Fig. 1.14 is described by the following equations:

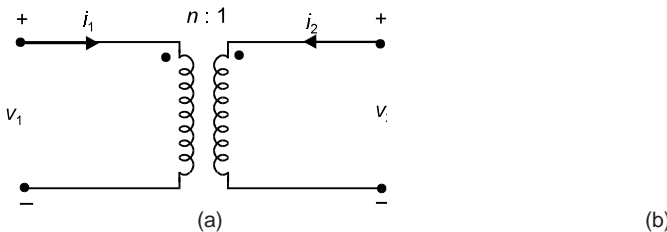


Fig. 1.14 The ideal transformer.

For Fig. 1.14 (a)

$$v_1 = n v_2 \quad \text{and} \quad i_2 = -n i_1 \quad \dots(1.50)$$

If the direction of i_2 is reversed as what is given in Fig. 1.14 (b), $i_2 = n i_1$

In matrix notation

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} \quad \dots(1.50)$$

$$\begin{aligned}
&= \int_{-\infty}^t \left(L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \right) i_1 dt + \int_{-\infty}^t \left(L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \right) i_2 dt \\
&= \int_0^{i_1} L_1 i_1 di_1 + \int_0^{i_1, i_2} M (i_2 di_1 + i_1 di_2) \\
&= \frac{1}{2} L_1 i_1^2 + M i_1 i_2 + \frac{1}{2} L_2 i_2^2 \quad \dots(1.54)
\end{aligned}$$

For perfect transformer to be passive, the energy stored should be non-negative i.e.

$$L_1 i_1^2 + 2 M i_1 i_2 + L_2 i_2^2 \geq 0 \quad \dots(1.55)$$

To prove that it is non-negative, we prove that the minimum of this is non-negative and hence the expression will be non-negative for all other values.

Let $i_2 = -xi_1$ where x is a real positive number.

The above expression reduces to

$$\begin{aligned}
f(x) &= L_1 i_1^2 - 2 M x i_1^2 + L_2 x^2 i_1^2 \\
\frac{d f(x)}{d x} &= -2 M i_1^2 + 2 L_2 x i_1^2 = 0
\end{aligned}$$

or
$$x = \frac{M}{L_2}$$

Substituting value of x in $f(x)$ we have

$$f(x) = \left(L_1 - 2 \frac{M^2}{L_2} + \frac{M^2}{L_2} \right) i_1^2 \geq 0$$

or
$$L_1 L_2 - M^2 \geq 0$$

or
$$M^2 \leq L_1 L_2$$

or
$$\frac{M^2}{L_1 L_2} \leq 1$$

Let
$$K^2 = \frac{M^2}{L_1 L_2} \quad \text{or} \quad K = \sqrt{\frac{M}{L_1 L_2}}$$

Since this inequality is true, the inequality (1.55) is also true.

Here K is known as co-efficient of coupling and its maximum value is unity when this transformer is known as a perfect or perfectly coupled transformer. It can be seen that a perfect transformer is not same as ideal transformer. To establish the difference we know

that in case of ideal transformer $\frac{v_1}{v_2} = n$

In case of perfect transformer

$$\frac{v_1}{v_2} = \frac{L_1 di_1/dt + M di_2/dt}{L_2 di_2/dt + M di_1/dt}$$